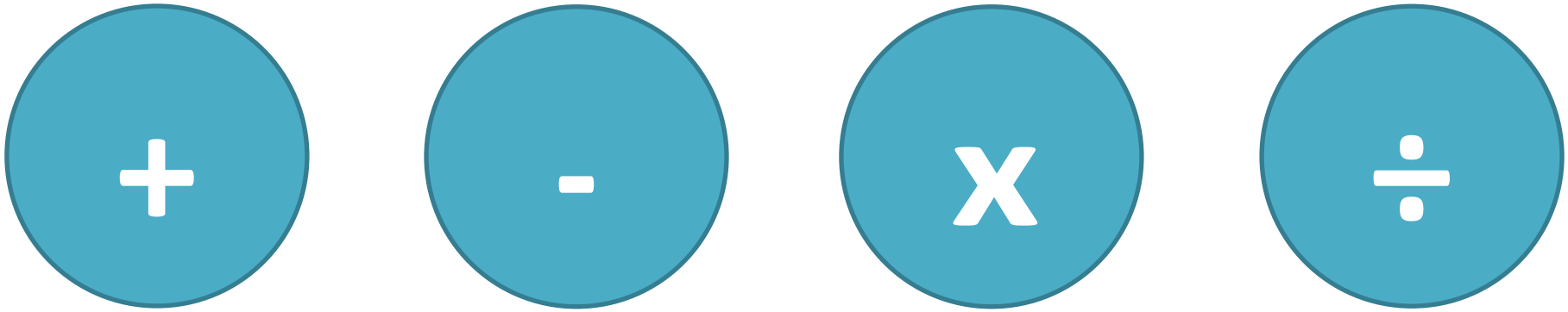
Summercourt Academy

Mathematics Calculation Guidance



Introduction

Welcome to the Summercourt Calculation Guidance! The purpose of this document is to create a new, updated guidance reflecting the requirements of the new curriculum and more importantly, the needs of our pupils based on knowledge and skills of those teachers working within our school and beyond.

This guidance aims to develop, model and explain core understandings and mathematical principles and progression to ensure consistency in the teaching and learning of mathematics in our school.

The focus of this guidance is the calculation of the four mathematical operations with an emphasis on written strategies to clarify processes and understanding and to make direct links to mental calculating. It is crucial that these mental strategies are discretely taught and linked to written strategies and not confined to starter activities in lessons. Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

The guidance shows clear steps towards achieving the end of year expectation as outlined by the National Curriculum in a progressive and scaffolded way by moving from concrete models and images to a final, perhaps more abstract representation of a mathematical calculation.

The overall aims of this guidance are that, when children leave our primary schools they:

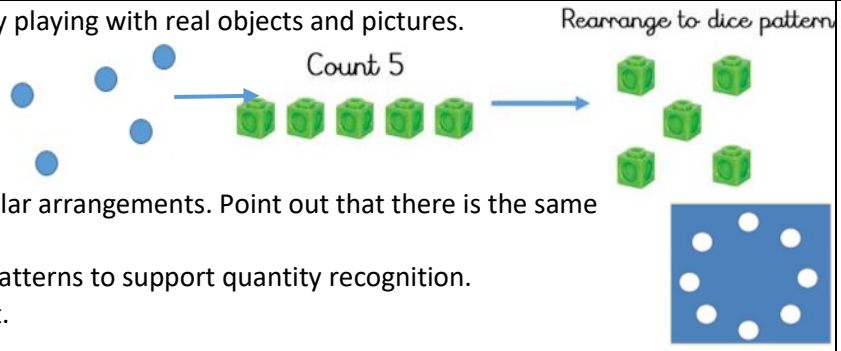
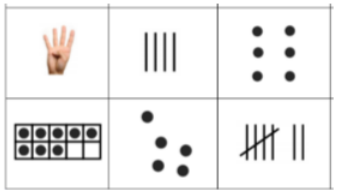
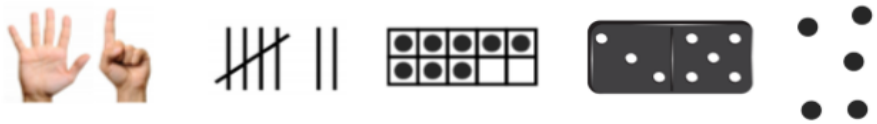
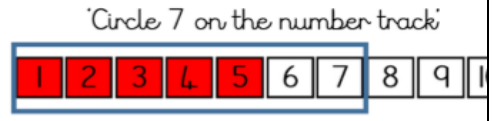
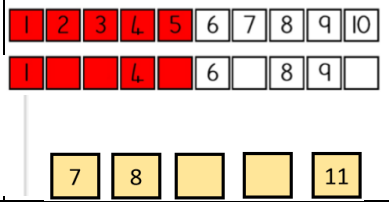
- ✓ have a secure knowledge of number facts and a good understanding of the four operations supported by a fluency and understanding of the fundamentals of mathematics;
- ✓ know the best strategy to use, estimate before calculating, systematically break problems down into a series of simpler steps with perseverance and use estimation and rounding to check that an answer is reasonable;
- ✓ can use these methods accurately with confidence and understanding;
- ✓ can use known facts in a variety of different contexts and apply the best strategy when problem solving
- ✓ make use of practical resources, diagrams and informal notes and jottings to help record steps and partial answers to support calculation before moving onto the abstract;
- ✓ have an efficient, reliable, compact written method of calculation for each operation, which they can apply with confidence when undertaking calculations;
- ✓ be able to identify when a calculator is the best tool for the task and use this primarily as a way of checking rather than simply a way of calculating;
- ✓ be able to explain their strategies to calculate and, using spoken language, give mathematical justification, argument or proof.

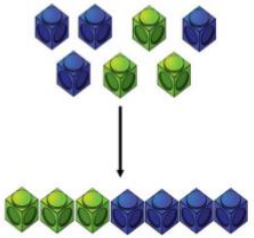
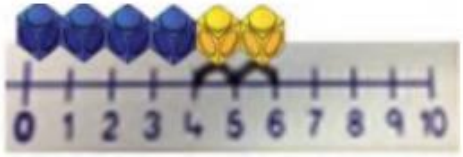
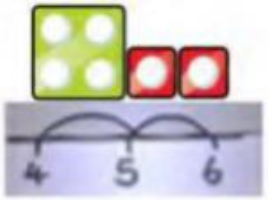
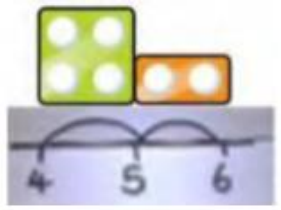
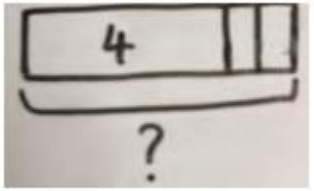
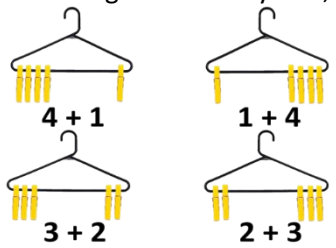
Statutory Requirements:

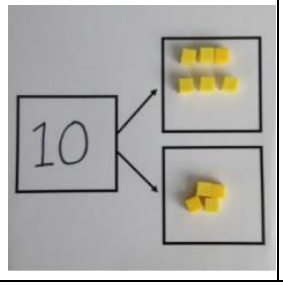
Early Learning Goal – Children should count reliably with numbers from one to 10, place them in order and say which number is one more or one less than a given number. Use quantities and objects, add and subtract two single-digit numbers and count on or back to find the answer. Count on from first group to add two groups of object.

Vocabulary

Plus, add, more, total, sum, altogether, make, parts and wholes, how many more is . . .?, 'is equal to', 'is the same as'

Objective	Concrete and Visual representations	Imagery → Abstract
Count reliably with numbers from 1-20	⊕ Children in EYFS first learn about addition by playing with real objects and pictures. ⊕ For 1:1 counting, number sounds are clearly separated and items counted with exaggerated movements. ⊕ Children learn that the last number is the total for the set— count small sets in irregular arrangements. Point out that there is the same number even though they are rearranged. ⊕ Counted objects are rearranged in regular patterns to support quantity recognition. ⊕ Progress by counting items from a larger set. ⊕ Count objects that can't be moved. 	⊕ Progress by making objects not visible once counted; ⊕ Count movements and sounds.
Identify and use numerals. Secure knowledge of numbers as quantities	⊕ Children match numerals to different representations of number for quantities 1-10 (e.g. making and finding 5 in different ways). ⊕ Children show numbers in different ways on fingers; ⊕ Quick recognition of regular and irregular dot patterns, with larger quantities visualised in two parts (e.g. see 5 as 3 and 2). Children are taught to recognise quantities on 10-frame and base-5 number track    ⊕ Instantly subitise 1-5 items through dot pattern games and everyday experiences. Items may be unrelated. ⊕ Children learn that 'teen' represents 10 and match teen/ten visual cards. 	⊕ Forwards and backwards number sequences supported using songs and rhymes. ⊕ Continue number sequences starting from different numbers e.g. 3, 4, 5, 6… ⊕ Number tracks used, with numbers hidden to add challenge as appropriate. 

<table>
<tr>
<td>Add single digit numbers</td>
<td>⊕ Addition built on experience of counting two groups of objects. Add groups by combining two parts to make a whole. Opportunities provided for comparing quantities, using language more/less.
⊕ Combining quantities in 10-frames and using Numicon encourage non-counting-in-ones strategies. Arrangement of sets counted also encourage counting on and calculation strategies.
 
     Representation of 4+3 encourages counting on from 4  Representation of 4+3 to help visualise 3+3+1</td>
<td> 
How many more is…?
Counting on taught by counting two sets, then screening 1 of the counted sets.</td>
</tr>
<tr>
<td>Record addition using + and = symbols</td>
<td>⊕ Children should begin to construct simple number sentences verbally and with pictures initially before moving onto formal recording.
 
    
⊕ Adding in jumps bigger than one.
   
8 + 1 = 9
⊕ Recognising that when adding small quantities only (1, 2 or 3), counting on from the greater is more efficient.
  </td>
<td> 
</td>
</tr>
<tr>
<td>Addition can be done in any order.</td>
<td>⊕ Investigate the story of 4, 5, 6, 7, 8, 9 and 10. E.g. <u>Partition</u> 5 into pairs and record the related additions.
 

Pupils could place the amount on top of the whole as well as writing the number down. The parts could also be written in alongside the concrete representation.
This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.
 
</td>
<td></td>
</tr>
</table>

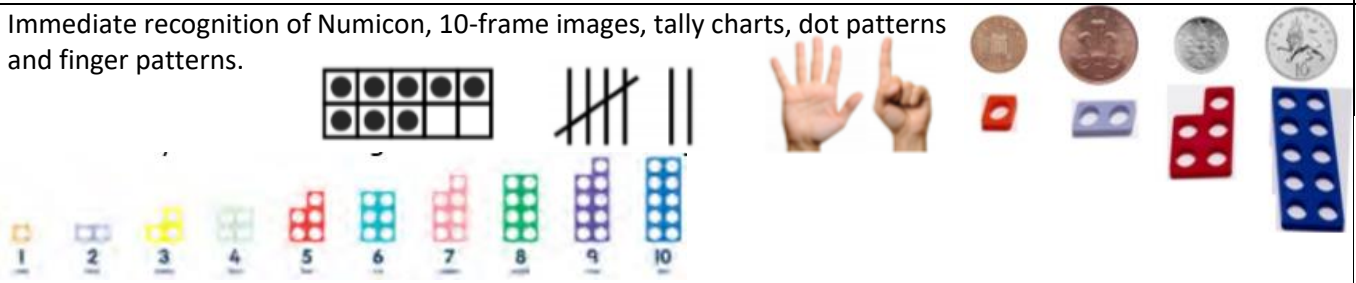
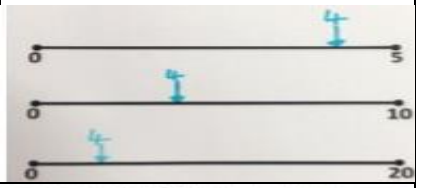
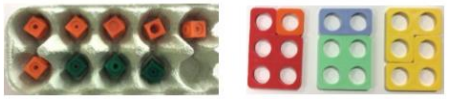
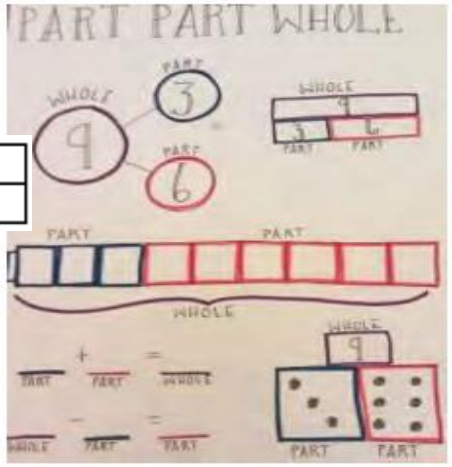
YEAR 1 **ADDITION**

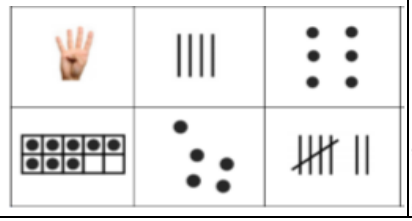
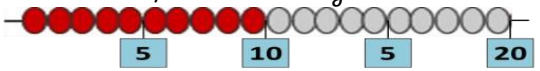
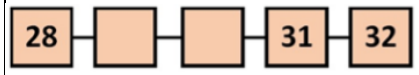
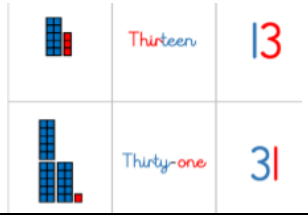
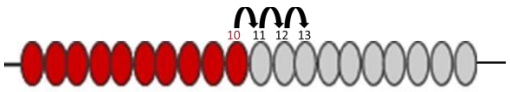
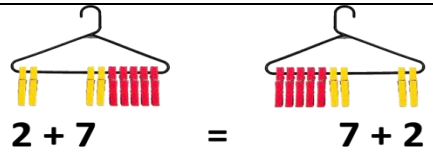
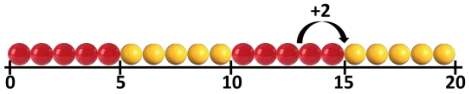
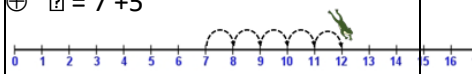
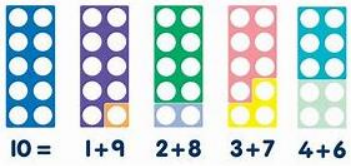
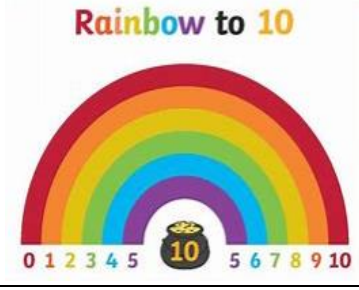
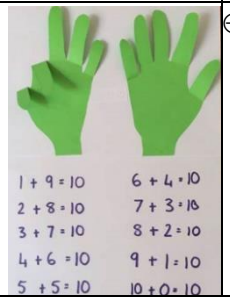
Statutory Requirements:

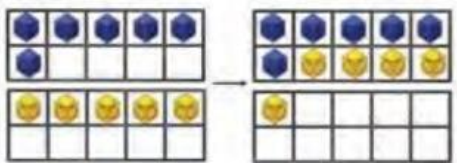
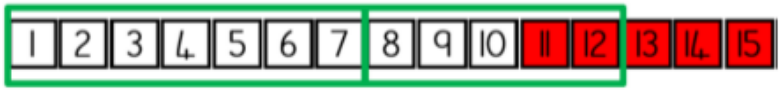
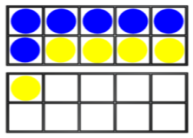
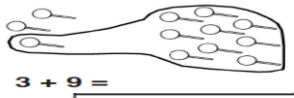
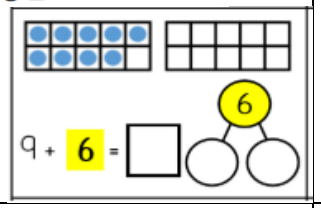
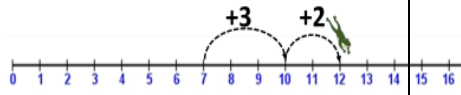
- ⊕ Read, write and interpret mathematical statements involving addition (+) and equals (=) signs – this means THE SAME AS – relate this to balance number sentences and scales
- ⊕ Represent and use number bonds and related subtraction facts within 20
- ⊕ Add one-digit and two-digit numbers to 20, including zero
- ⊕ Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 9 = □ + 7

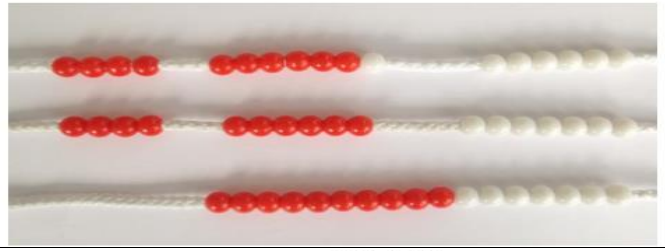
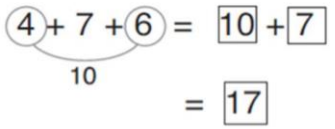
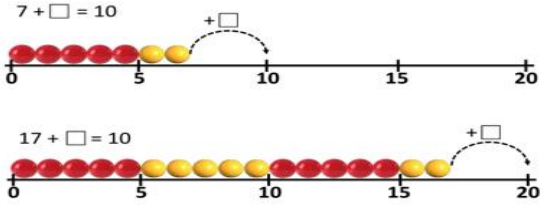
Vocabulary

Plus, add, more, total, sum, altogether, make, **partition,** parts and wholes, how many more is . . .?, **tens, ones, teen number,** 'is equal to', 'is the same as', **number bonds, number line, hundred squares, inverse, double, near double**

Objective	Concrete and Visual representations	Imagery → Abstract
Represent 1-10 in different ways, subitising small quantities, without counting all items	Immediate recognition of Numicon, 10-frame images, tally charts, dot patterns and finger patterns. 	Estimate position of numbers on blank number lines with different start/end numbers. 
Partition 1-10 in all possible ways, write number sentences using + and =	⊕ Subitizing regular and irregular dot patterns, with children visualising quantities in two parts  ⊕ Arrangement of 2 colours of items e.g. in egg box 10-frame or with Numicon. 	⊕ Introduction of part-whole model from individuals squares/items to bars.  

Know 1 more/less in the range 1-100, **<u>focusing on bordering tens boundaries</u>**	⊕ Primarily children will work with numbers within 20 as they learn to confidently cross the ten boundary, progressing to numbers beyond 20. ⊕ Identify and show one more/less in different ways. Example game: one more/less bingo. 	⊕ Use landmarks of 5s to help place other numbers on a washing line or bead bar. Where is 11, 13? How do you know?  	Find missing numbers on number track, focusing on tens boundaries. 
Recognise tens and ones in 2-digit numbers	⊕ Count on from 10 to make **teen** numbers. 10 + 3 = ? ⊕ Organise large quantities in groups of 10 and ones. Partition 2-digits numbers using dienes, place-value cards.    		 Using place value cards to represent 2 digit numbers. Understand what each digit represents.
Add 2 numbers by counting on (using objects, then using fingers)	⊕ Adding can be done in any order. Demonstrate symbolism of balanced calculations and commutative number sentences. Show equals sign can be use on either side of a calculation (? = 7+5).  ⊕ As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater is more efficient. E.g. 15 = 12 + 3   ⊕ 13 + 2 =  	⊕ What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2?  2 + 4 = 4 + 2 = ? ⊕ □ = 7 +5  ⊕ Progress to count on from the greater number, using fingers to tract the jumps.
Know number bonds to 10	⊕ Identify patterns of the number bonds using various visual representations (such as fingers, Numicons number line or bead bar, bar models)    	⊕ Memorise number bonds to 10. E.g. 6 + □ = 10

<table>
<tr>
<td>Add 2 single digit numbers bridging 10 by regrouping <u>10 ones to make 1 ten</u></td>
<td>⊕ Rather than counting on as their main strategy, pupils should be encouraged to rely on number bonds knowledge as time goes on.
Use 10 frames, Dienes and 2-colour number tracks show when 10 ones can be regroup to make 1 ten: ‘how many to 10, how many more to add?’
6 + 5
 
7 + 5 =
 
9 + 3 =
  </td>
<td>⊕ Children then move on to draw ten frames and dots for counters.
 
Or they can draw objects then group 10 ones to make 1 ten.
 
3 + 9 =
 
</td>
<td>⊕ Progress to use of blank number line. 7 + 5 =
 

⊕ Develop an undestanding of equality. 7 + 3 + 2 = 10+2
? = 4 + 3 + 6
6 + □ = 11
6 + 5 = 5 + □
6 + 5 = □ + 4</td>
</tr>
<tr>
<td>Add 3 single digit numbers, (make 10 first).</td>
<td>  </td>
<td colspan="2">Pupils may need to try different combinations before they find the two numbers that make 10.
The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.
The second bead string shows 4, 6 and then 7.
The final bead string shows how they have now been put together to find the total.</td>
<td>Reinforce balancing number sestence and = means is the same as
 
</td>
</tr>
<tr>
<td>Find compliment to the next ten.</td>
<td>⊕ Adding to the next tens: Use number bonds to 10 to help,
⊕ or counting on on 1-100 number grid (jumping frog)
e.g. 36 + 4 =
45 + ? = 50
23 + ? = 30
 

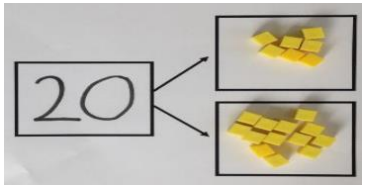
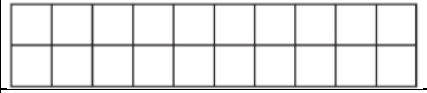
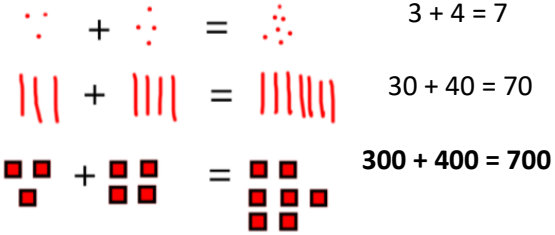
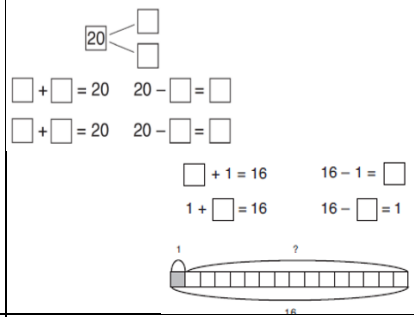
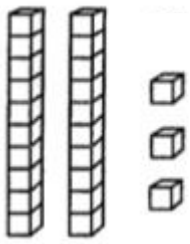
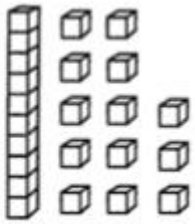
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40</td>
<td>⊕ Adding 10s sliding up and down like a ‘Spider’
23 + 10 =
23 + 20 =</td>
<td colspan="2">1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100</td>
</tr>
<tr>
<td>Applying</td>
<td colspan="4">⊕ Solve one-step problems that can involve addition and subtraction, using concrete objects and pictorial representations
⊕ Compare, describe and solve practical problems for:
– Lengths and heights (e.g. long/short, longer/ shorter, tall/ short, double/half)
– Mass or weight (e.g. heavy/light, heavier than, lighter than)
– Capacity/ volume (full/empty, more than, less than, quarter) and – Time (quicker, slower, earlier, later)</td>
</tr>
</table>

Statutory Requirements:

- ⊕ Solve problems with addition using concrete objects and pictorial representations, including those involving numbers, quantities and measures, apply their increasing knowledge of mental and written methods.
- ⊕ Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- ⊕ Add numbers using concrete objects, pictorial representations and mentally, including:
  - a two-digit number and ones.
  - a two-digit number and tens,
  - two two-digit numbers
  - three one-digit numbers
- ⊕ Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- ⊕ Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

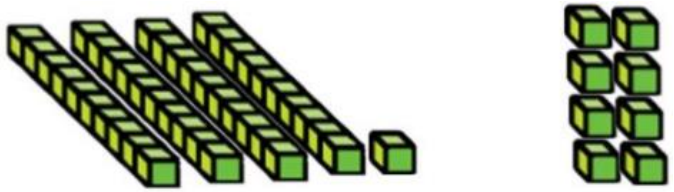
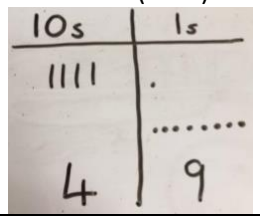
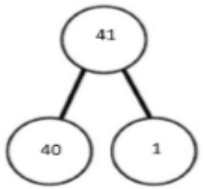
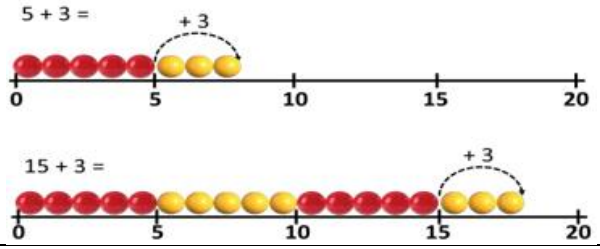
Vocabulary

Plus, add, more, total, sum, altogether, make, partition, **recombine,** parts and wholes, how many more is . . .?, digit, **hundreds**, tens, ones, 'is equal to', 'is the same as', number bonds, number line, inverse, double, **near multiples, commutative law**

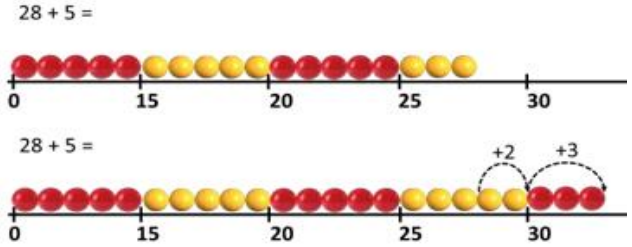
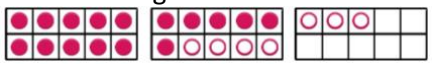
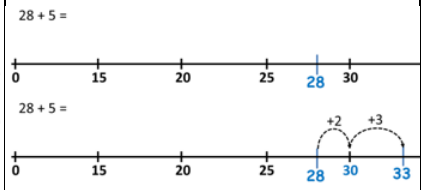
Objective	Concrete and Visual representations		Imagery → Abstract
Number facts and Parts Whole model (teach + and – together)	Building on work done in Year 1, **revise number bonds to 10 and the inverse**. Progress to bonds to 100   20 = 7 + 13 20 = 13 + 7 7 + 13 = 20 13 +7 = 20 **Addition is commutative.** 20 – 7 = 13 20 – 13 = 7  3 + 4 = 7 30 + 40 = 70 **300 + 400 = 700**		
Represent number 1-100 in different ways, showing understanding of place value	Represent ten/teens using Dienes, Numicon, ten frames showing numbers in different ways.  	⊕ Partition 2 digit numbers using place value cards.  ⊕ Draw images show a number in different ways using base 10 ⊕ Find missing number on 100 squares	⊕ Estimate position of numbers on blank number lines with different start/end numbers. 

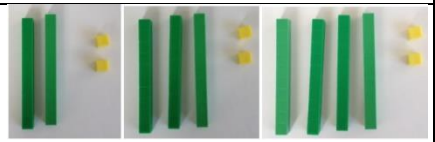
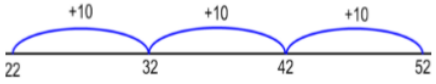
<table>
<tr>
<td rowspan="2">Add 1 digit number with a 2 digit number.

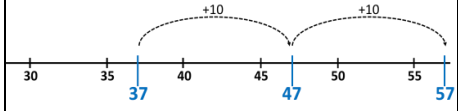
TO + O

<u>Not crossing 10s.</u></td>
<td>⊕ Using Dienes 41 + 8 =
  </td>
<td>Draw tens (lines) and ones (dots)
 
</td>
<td>41 + 8 1 + 8 = 9
40 + 9 = 49
 
</td>
</tr>
<tr>
<td>⊕ Mental strategy
Counting on from the larger number.
 
</td>
<td colspan="2">→ Progress to using number facts & recognising patterns and the children won't need to count on.
5 + 3 = 25 + 3 =
15 + 3 = 35 + 3 = …
95 + 3 =</td>
</tr>
<tr>
<td>Add 1 digit number with a 2 digit number.

TO + O
<u>Crossing 10s</u>.

(make the next multiple of 10 first)</td>
<td>16 + 7 = 16 + 4 + 3 = 20 + 3 = 23
 
Use other representations such as Dienes, Beads strings:
 
</td>
<td>Draw ten frames and dots to show the partitioning of the 1 digit numbers to compliment the 2 digit number to make a multiple of 10 before adding.
 
Draw Dienes:
 
'Ten ones make 1 ten'</td>
<td>⊕ Progress to use of blank number line
 
</td>
</tr>
<tr>
<td>Add multiples of ten

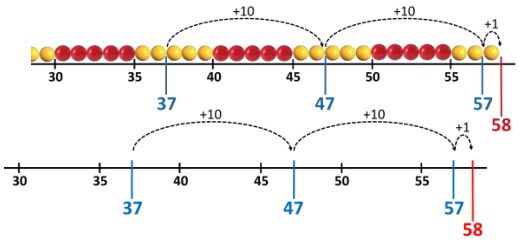
TO + 10s</td>
<td> 
⊕ 2 tens + 3 tens = 5 tens
20 + 30 = 50
⊕ Derive number bonds to 100 using number bonds to 10.
2 + 8 = 10 → 20 + 80 = 100 etc.</td>
<td> 
22 + 30 (32, 42, 52)
 

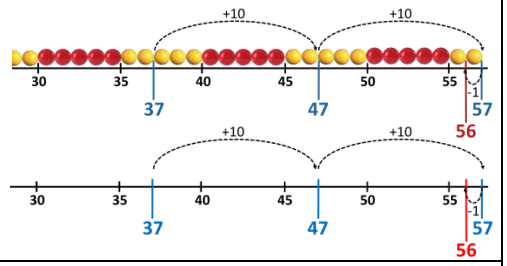
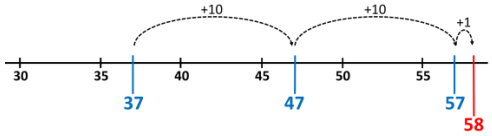
Recognise that the ten digit changes but the one digit doesn't change.</td>
<td>⊕ Draw Dienes: 43 + 20 =
Count in 10s: 43, 53, 63
 
⊕ Draw 2 jumps of 10 on a number line, labeling the jumps as couniting.
 
</td>
<td>43 + 20 = 63 (spider)
43 + □ = 83
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr>
<tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr>
<tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr>
<tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr>
<tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr>
<tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr>
<tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr>
</table></td>
</tr>
</table>

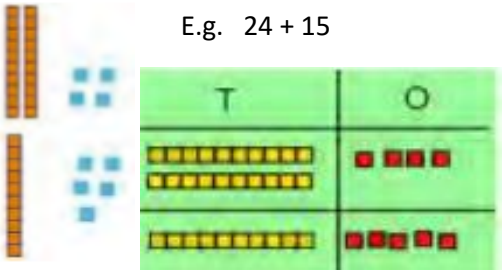
<table>
<tr>
<td>Mental strategy
Add near multiples of 10

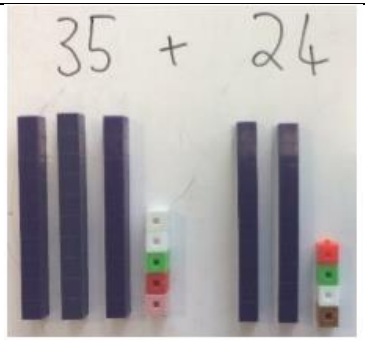
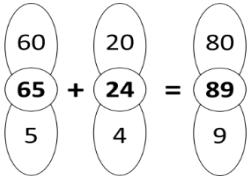
TO + 9, 19
TO + 11, 21</td>
<td>Investigate different strategies to + 9, + 19, +29 or +11, +21, +31
Arrive at the most efficient method, which is by adding multiples of ten and then adjust.

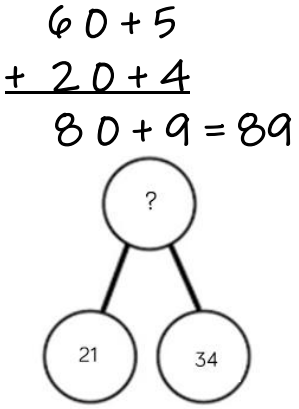
Children should make observation on what happen to each digit after adding 9 or adding 11 and able to describe this observation.

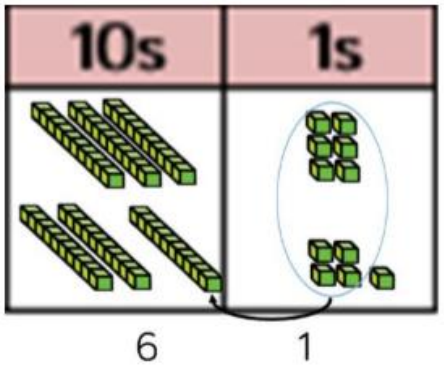
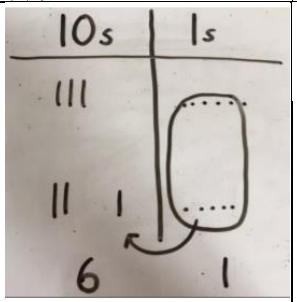
37 + 21
 

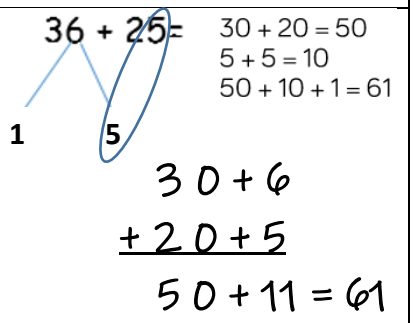
37 + 19
 
</td>
</tr>
<tr>
<td>Mental strategy
Counting on, partitioning the 2nd number only</td>
<td>Similar as above. 37 + 22
Partition only 22 = 20 + 2.
Count on 2 tens and 2 ones
 
</td>
</tr>
<tr>
<td>Add two 2-digit numbers by partitioning

TO + TO
<u>(without exchanging/ regrouping)</u></td>
<td>⊕ Use concrete apparatus to represent each 2 digit numbers.
Then count all the tens together and count all the ones together.
Recombine to find the answer.
(5 tens + 9 ones = 59)
⊕ Progress to children drawing images of tens and ones
E.g. 24 + 15
 

 
</td>
<td>⊕ <u>Petal method</u> (an alternative method to demonstrate partitioning): Use visual images of tens and ones in each petal for scaffolding.
 

⊕ Alternatively, use place value cards to demonstrate partitioning
65 + 24 = 60 + 20 + 5 + 4
= 80 + 9
= 89</td>
<td> 
</td>
</tr>
<tr>
<td>Add two 2-digit numbers by partitioning

TO + TO
<u>(with exchanging/ regrouping)</u></td>
<td>⊕ Regroup 10 ones to make 1 ten.
 
</td>
<td>⊕ Children draw tens and ones.
 

indicating ten ones make 1 tens!</td>
<td> 
</td>
</tr>
</table>

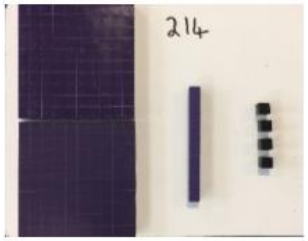
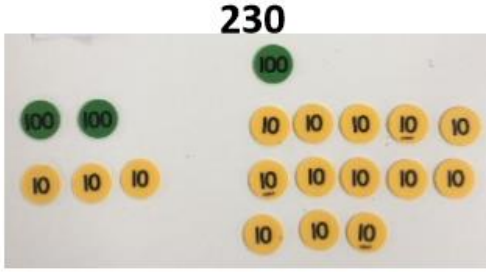
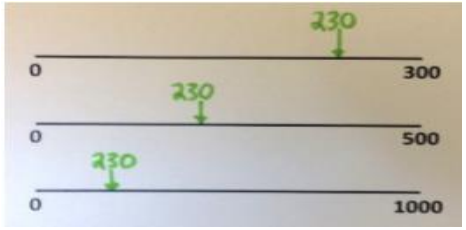
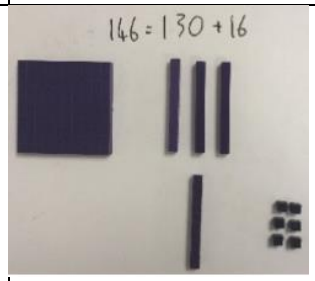
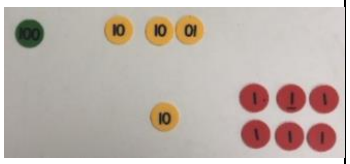
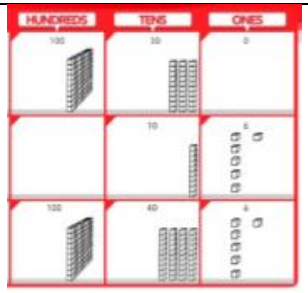
Statutory Requirements:

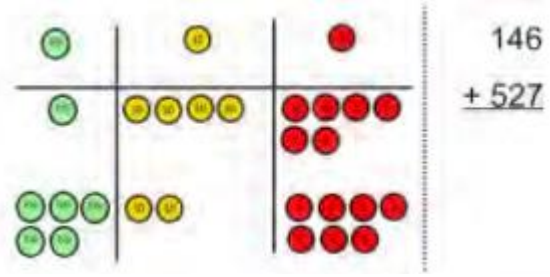
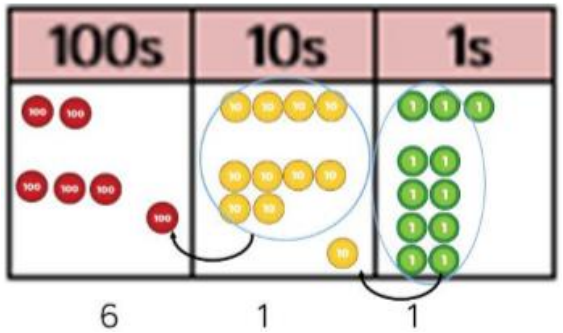
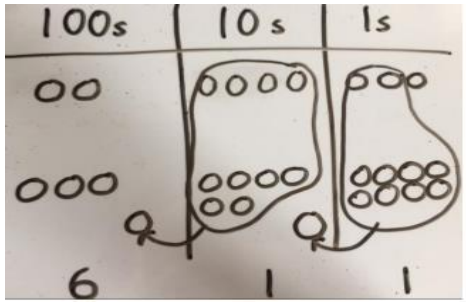
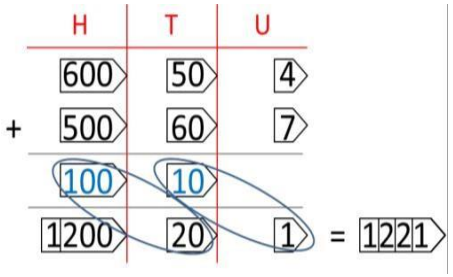
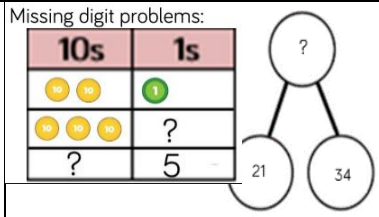
- ⊕ Add numbers mentally, including:
  - a three-digit number and ones,
  - a three-digit number and tens
  - a three-digit number and hundreds
- ⊕ Add numbers with up to three digits, using formal written methods of column addition
- ⊕ Estimate the answer to a calculation and use inverse operations to check answers
- ⊕ Solve problems, including missing number problems, using number facts, place value, and more complex addition.

Vocabulary

Plus, add, more, total, sum, altogether, make, partition, recombine, how many more is . . .?, hundreds , 'is equal to', 'is the same as', digit, inverse, **column addition, vertical, 'regroup', expanded, compact**

Building on work done in Year 2, it is crucial that children have a secure understanding of place value as they move to more formal calculation strategies for addition; continuous checks and references should be made.

Objective	Concrete and Visual representations		Imagery → Abstract
Represent 3digit numbers in a range of ways, showing an understanding of place value	Make 3-digit numbers using dienes and place value cards, showing how they can be partitioned.  	Make the same number in different ways with place value counters. 	Estimate position of numbers on blank number lines with different start/end numbers. 
Expanded column method HTO + TO, HTO + HTO (without exchanging)	 	 Children draw 100s, tens and ones to demonstrate partitioning and adding.	100 + 30 + 10 + 6 = 100 + 40 + 6 = 146

Expanded method. HTO + HTO <u>with a single exchange (10 or 100)</u>	 146 + 527	Chidlren draw PV counters as below **or** using PV cards to support the recording of the expanded method. Make realistic estimation by rounding. 673 = 600 + 70 + 3 100 + 40 + 6 500 + 20 + 7 600 + 60 + 13 10 3	H T O 1 4 6 + 5 2 7 1 3 6 0 6 0 0 6 7 3 2 3 6 + 7 3 9 1 0 0 2 0 0 3 0 9
Compact method. HTO + HTO <u>with more than 1 exchanges (10/100/1000)</u>	**243 + 368**  100s 10s 1s 6 1 1 Use PV counters to partition each numbers into PV grid. Model adding the O(1s), T(10s) and H(100s) in that order (recording in the correct column and emphasising the value of the numbers being added). REMEMBER TO ADD THE VALUE 'EXCHANGED' WHEN BRIDGING 10/100, WHEN ADDING THE RELEVANT COLUMN. Check the anwer by rounding.	**243 + 368** Chidren to represent the counters in a place value chart, circling when they make an exchange.  100s 10s 1s 6 1 1 **654 + 567** Draw PV counters or using PV cards to support the recording of formal written method.  H T U 600 50 4 + 500 60 7 100 10 1200 20 1 = 1221	H T O 243 +368 611 1 1 H T O 6 5 4 + 5 6 7 12 2 1 = 1221 1 1
Applying Conceptual Variation	 21 +34 21 + 34 = □ = 21 + 34	⊕ In year 3, there are 21 children and in year 4, there are 34 children. How many children in total? ⊕ 21 + 34 = 55. Prove it ⊕ Calculate the sum of twenty-one and thirty-four.	Missing digit problems:  10s 1s ? 5 ? 21 34 ? 21 34

YEAR 4	ADDITION

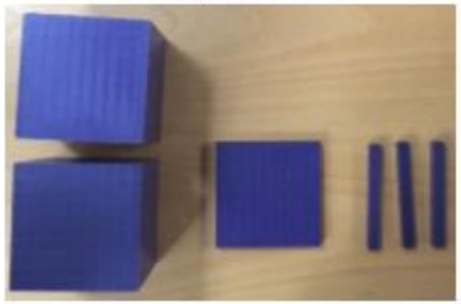
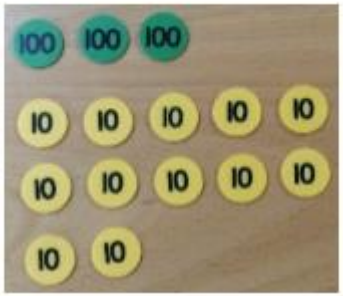
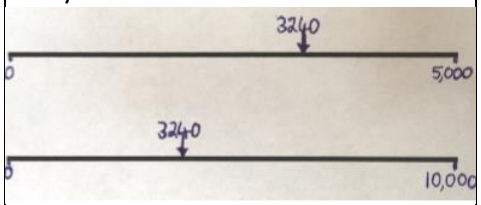
Statutory Requirements:

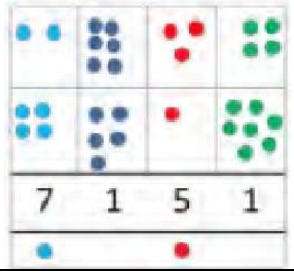
- ⊕ Add with up to 4 digits using the formal written methods of column addition where appropriate
- ⊕ Estimate and use inverse operations to check answers to a calculation
- ⊕ Solve addition two-step problems in contexts, deciding which operations and methods to use and why

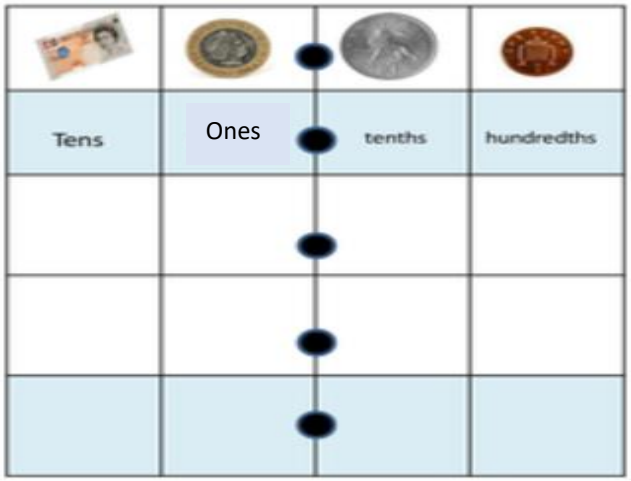
Vocabulary

Plus, add, more, total, sum, altogether, make, partition, recombine, how many more is . . .?, **thousands,** hundreds, tens, ones, 'is equal to', 'is the same as', inverse, column addition, vertical, 'regroup', expanded, compact, number line, **increase**, digits, **tenths, hundredths, decimal (places),** count through zero

Building on work done in Year 3, it is crucial that children have a secure understanding of place value; It may be appropriate to revisit the methods taught in Y3 as a starting point.

Objective	Concrete and Visual representations		Imagery → Abstract
Represent 4digit numbers in a range of ways, showing an understanding of place value	Make 4-digit numbers using dienes and place value counters, showing how they can be partitioned. 2,130 	Make the same number in different ways with place value counters. 420 with three 100s and twelve 10s 	Estimate position of numbers on blank number lines with different start/end numbers. 
Choose efficient mental strategies for adding numbers	Round and adjust to calculate, 350+98= Model with appropriate visual (place value counters). Add 100, take away 2.	3999 + 1001 = "Is a column method the best strategy? Why?"	

<table>
<tr>
<td>Expanded and compact column addition with 4 digit numbers, <u>(regrouping 10/100)</u></td>
<td>2634 + 4517 = 2634 + 4517 (commutative)
 
7 1 5 1
Children can also do the same process when adding 3 numbers.</td>
<td>Use PV cards or drawing:
<table>
<tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2000</td><td>600</td><td>30</td><td>4</td></tr>
<tr><td>+</td><td>4000</td><td>500</td><td>10</td><td>7</td></tr>
<tr><td></td><td>7000</td><td>1100</td><td>50</td><td>11</td></tr>
<tr><td></td><td>1000</td><td></td><td>10</td><td></td></tr>
</table>
</td>
<td>
<table>
<tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr>
<tr><td></td><td>2</td><td>6</td><td>3</td><td>4</td></tr>
<tr><td>+</td><td>4</td><td>5</td><td>1</td><td>7</td></tr>
<tr><td></td><td>7</td><td>1</td><td>5</td><td>1</td></tr>
<tr><td></td><td>1</td><td></td><td>1</td><td></td></tr>
</table>
</td>
</tr>
<tr>
<td>Expanded and compact column addition with decimal in money context

<u>(including bridging 10p, £1)</u></td>
<td>£3.24 + £2.58 =
Chilren use coins to place on the place value chart.
 

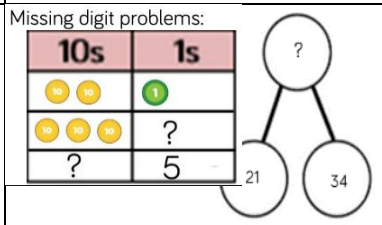
Decimal point does not move!
Limit the amount of carrying initially so that the process is clear.</td>
<td>
<table>
<tr><th>£1</th><th>10p</th><th>1p</th></tr>
<tr><td>3</td><td>20</td><td>4</td></tr>
<tr><td>+ 2</td><td>50</td><td>8</td></tr>
<tr><td>5</td><td>80</td><td>2 = £5.82</td></tr>
<tr><td></td><td>10</td><td></td></tr>
</table>

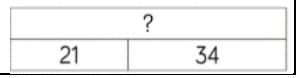
<table>
<tr><th>£1</th><th>10p</th><th>1p</th></tr>
<tr><td>3</td><td>70</td><td>4</td></tr>
<tr><td>+ 2</td><td>80</td><td>7</td></tr>
<tr><td>6</td><td>60</td><td>1 = £6.61</td></tr>
<tr><td>1</td><td>10</td><td></td></tr>
</table>
</td>
<td>
<table>
<tr><td>£3</td><td>2</td><td>4</td></tr>
<tr><td>£2</td><td>5</td><td>8</td></tr>
<tr><td>£5</td><td>8</td><td>2 = £5.82</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

<table>
<tr><td>£3</td><td>7</td><td>4</td></tr>
<tr><td>£2</td><td>8</td><td>7</td></tr>
<tr><td>£6</td><td>6</td><td>1 = £6.61</td></tr>
<tr><td>1</td><td>1</td><td></td></tr>
</table>
</td>
</tr>
<tr>
<td>Applying

Conceptual Variation</td>
<td> 
</td>
<td>⊕ In KS1, there are [3digit number] books and in KS2, there are [3d number] books. How many books in total?

⊕ 121 + 334 = 455. Prove it

⊕ Calculate the sum of HTO and HTO.</td>
<td>Missing digit problems:
 

 
</td>
</tr>
</table>

YEAR 5	ADDITION

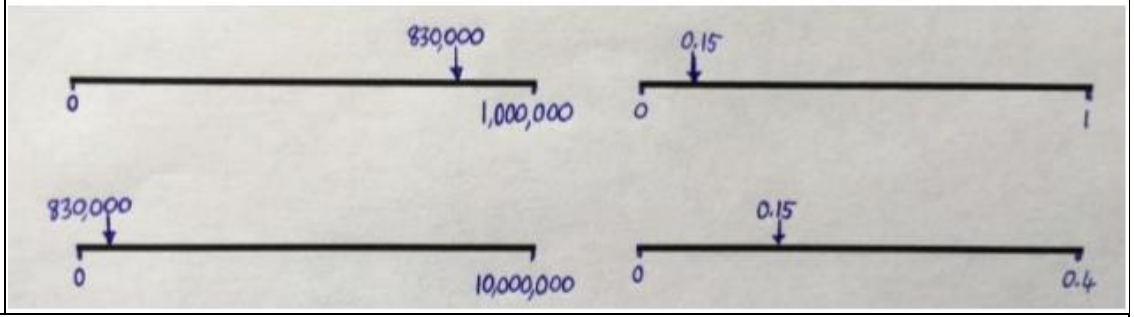
Statutory Requirements:

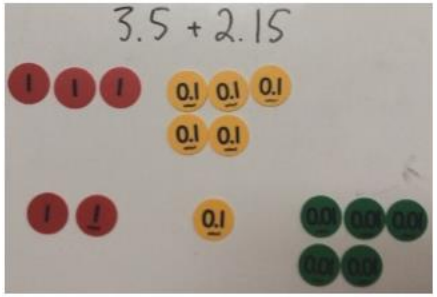
- ⊕ Add whole numbers with more than 4 digits, including using column addition where appropriate
- ⊕ Add numbers mentally with increasingly large numbers
- ⊕ Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- ⊕ Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Vocabulary

Plus, add, more, total, sum, altogether, make, partition, recombine, how many more is . . .?, **ten thousands,** thousands, hundreds, tens, ones, 'is equal to', 'is the same as', inverse, column addition, vertical, 'regroup', expanded, compact, number line, increase, digits, tenths, hundredths, decimal (places), count through zero, **efficient written method**

Building on work done in Year 4, it is crucial that children have a secure understanding of place value; It may be appropriate to revisit the methods taught in Y4 as a starting point.

Objective	Concrete and Visual representations	Imagery → Abstract
Represent the value of digits in numbers of up to 7-digits and decimals to thousandths	Make numbers in the range using place value counters, showing the same number in different ways. 0.35  430 	Estimate position of numbers on blank number lines with different start/end numbers. 
Choose efficient strategies	Mental calculation methods modelled using appropriate visual, e.g. rounding and adjusting on a number line, bar model 235 + 2100.1	

<table>
<tr>
<td>Formal method
Add 2 decimal numbers with up to 2 decimal places. <u>(with regrouping)</u></td>
<td>Using place value counters
 
</td>
<td>
<table>
<tr><td></td><td>£10</td><td>£1</td><td>.</td><td>10p</td><td>1p</td></tr>
<tr><td></td><td>10</td><td>4</td><td>.</td><td>20</td><td>9</td></tr>
<tr><td>+</td><td>10</td><td>7</td><td>.</td><td>40</td><td>9</td></tr>
<tr><td></td><td>10</td><td></td><td>.</td><td>10</td><td></td></tr>
<tr><td></td><td>30</td><td>1</td><td>.</td><td>70</td><td>8</td></tr>
</table>
= £31.78

Check the answer by rounding.</td>
<td>
<table>
<tr><td></td><td>£1</td><td>4</td><td>2</td><td>9</td></tr>
<tr><td>+</td><td>£1</td><td>7</td><td>4</td><td>9</td></tr>
<tr><td></td><td>£3</td><td>1</td><td>7</td><td>8</td></tr>
<tr><td></td><td></td><td>1</td><td>1</td><td></td></tr>
</table>
= £31.78
<table>
<tr><td></td><td>£1</td><td>9</td><td>0</td><td>1</td></tr>
<tr><td></td><td>£</td><td>3</td><td>6</td><td>5</td></tr>
<tr><td>+</td><td>£</td><td>0</td><td>7</td><td>0</td></tr>
<tr><td></td><td>£2</td><td>3</td><td>3</td><td>6</td></tr>
<tr><td></td><td></td><td>1</td><td>1</td><td></td></tr>
</table>
With a more complex decimal calculation draw attention to the role of <u>0 as place holder</u> to ensure a clear understanding of place value.</td>
</tr>
<tr>
<td>Formal method
Compact addition <u>(with regrouping 10/100/1000/ 10,000)</u></td>
<td>35,272 + 28,345 =
<table>
<tr><td></td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>30 000</td><td>5000</td><td>200</td><td>70</td><td>2</td></tr>
<tr><td>+</td><td>20 000</td><td>8000</td><td>300</td><td>40</td><td>5</td></tr>
<tr><td></td><td>10 000</td><td></td><td>100</td><td></td><td></td></tr>
<tr><td></td><td>60 000</td><td>3000</td><td>600</td><td>10</td><td>7</td></tr>
</table>
Check the answer by rounding.</td>
<td>
<table>
<tr><td></td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td>3</td><td>5</td><td>2</td><td>7</td><td>2</td></tr>
<tr><td>+</td><td>2</td><td>8</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td></td><td>6</td><td>3</td><td>6</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>1</td><td>1</td><td></td><td></td></tr>
</table>
= 63,617
When working with larger numbers, model the <u>correct placement of the comma</u>. Check children can confidently <u>read these numbers</u>.</td>
</tr>
</table>

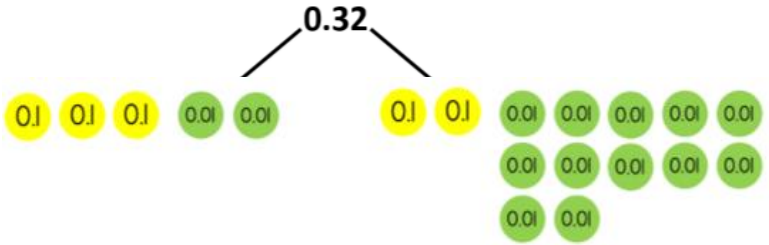
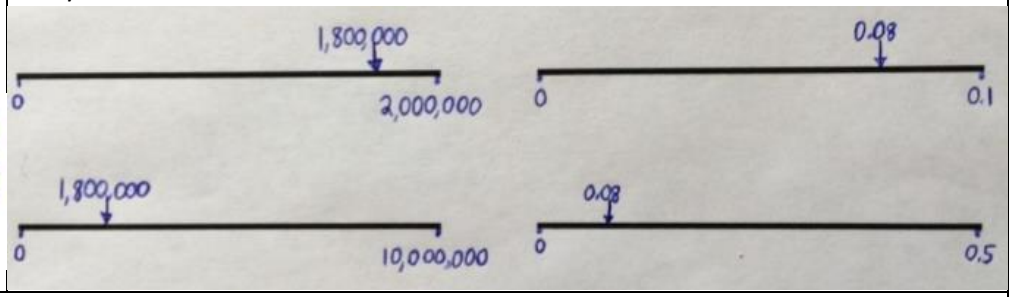
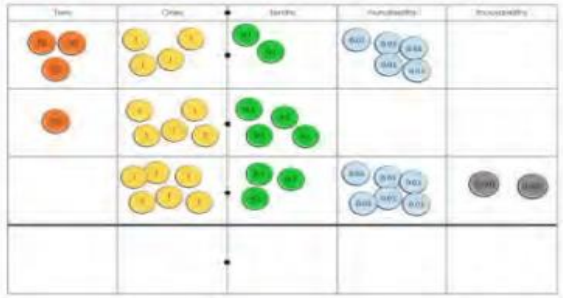
YEAR 6 — ADDITION

Statutory Requirements:

- ⊕ Perform mental calculations, including with mixed operations and large numbers
- ⊕ Use knowledge of the order of operations to carry out calculations involving the 4 operations
- ⊕ I can solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- ⊕ I can use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Vocabulary

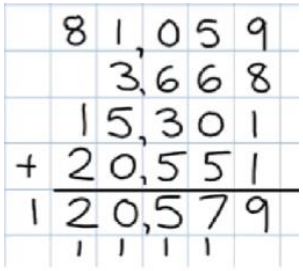
Plus, add, more, total, sum, altogether, make, partition, recombine, how many more is . . .?, **hundred thousands,** ten thousands, thousands, hundreds, tens, ones, 'is equal to', 'is the same as', inverse, column addition, vertical, 'regroup', expanded, compact, number line, increase, digits, tenths, hundredths, decimal (places), count through zero, efficient written method, **order of operations.**

<table>
<tr><th>Objective</th><th>Concrete and Visual representations</th><th>Imagery → Abstract</th></tr>
<tr><td>Represent the value of digits in numbers of up to 8-digits and decimals to thousandths</td><td>Make numbers in the range using place value counters, showing the same number in different ways.
 
</td><td>Estimate position of numbers on blank number lines with different start/end numbers.
 
</td></tr>
<tr><td>Choose efficient strategies</td><td colspan="2">Mental calculation methods modelled using appropriate visual, e.g. rounding and adjusting on a number line, bar model
235 + 400,004.1</td></tr>
<tr><td>Add several numbers with different numbers of decimal places (including money and measures)</td><td>Using place value counters
  </td><td>3 4 . 2 5 0
1 5 . 4 0 0
+ 6 . 3 6 2
5 6 . 0 1 2
1 1 1

Check the answer by rounding.

With a more complex decimal calculation draw attention to the role of <u>0 as place holder</u> to ensure a clear understanding of place value.</td></tr>
<tr><td>Adding several numbers with more than 4 digits.
<u>(including regrouping 10/100/1000/10,000/100,000)</u></td><td>Concrete and Visual images may still needed for scaffolding.
Children may use PV grid with PV counters.
 
</td><td>1 2 0 5 3 7
2 3 4 2 7 1
3 2 3 2 2 1
+
6 7 8 0 2 9
1 1

Check the answer by rounding.

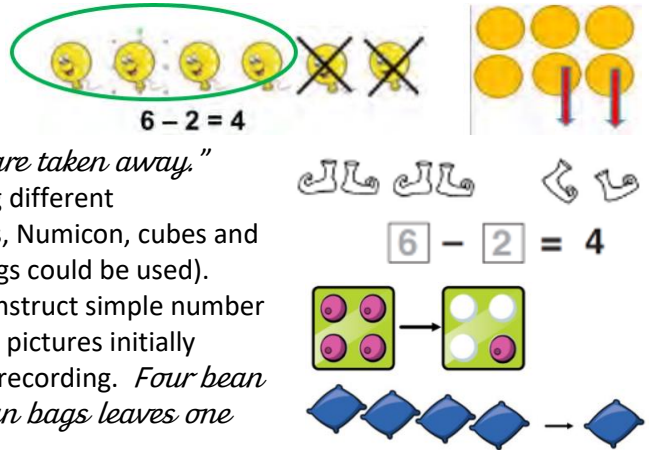
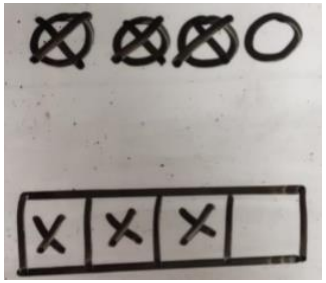
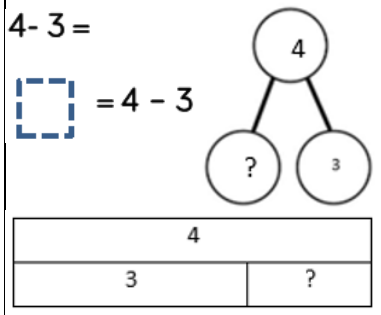
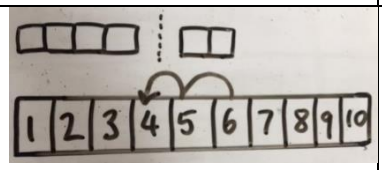
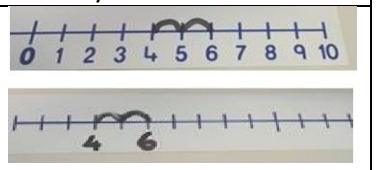
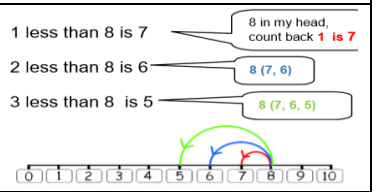
When working with larger numbers, model the <u>correct placement of the comma</u>. Check children can confidently <u>read these numbers</u>.
 
</td></tr>
</table>

Statutory Requirements:

Early Learning Goal – Children should count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Use quantities and objects, add and subtract two single digit numbers and count on or back to find the answer.

Vocabulary

take away, less than, one less, two less . . the difference between, subtract, minus, fewer, decrease, 'is equal to', 'is the same as', leave, how many are left/left over? how many have gone?

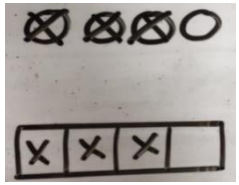
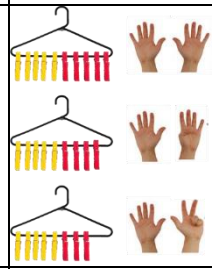
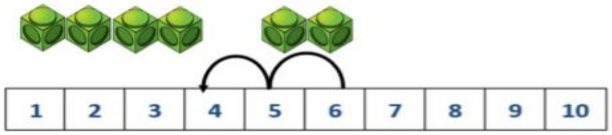
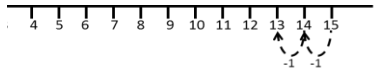
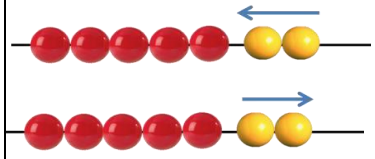
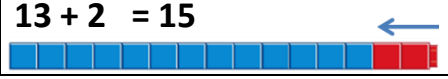
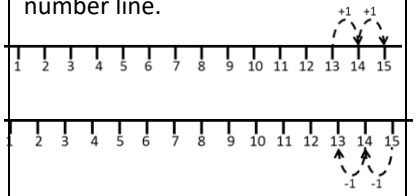
Objective	Concrete and Visual representations	Imagery	Abstract
Take away using objects	– Children first learn about subtraction by playing with real objects and pictures. – Physically taking away and removing objects from a whole (e.g whole group of 6). "*Count how many are left when two are taken away.*" – Represent subtraction using different representations (ten frames, Numicon, cubes and other items such as beanbags could be used). – Children should begin to construct simple number sentences verbally and with pictures initially before moving onto formal recording. *Four bean bags take away three bean bags leaves one bean bag. 4 – 3 = 1* 	– Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. *4 – 3 = 1* 	Part-whole model. 4- 3 = □ =4 – 3  Use this model to break down all number 1-10. Write number sentences using +, - and = symbols.
Take away (a small amount) by counting back	– Practice counting backwards from 10 through song and nursery rhymes. – Counting back (using bead string, number lines or number tracks) children start with 6 and count back 2. Record number sentence using -, = 6 – 2 = 4 	 Children to represent what they see pictorially	 Show the backwards jumps on a number line.
Find 1 less, 2 less, 3 less…	– Show subtractions in various context, using different vocabulary. *Maria has 3 teddies, Anna has 2 less.* – *Show me on your finger 2 less than 10. D*emonstrate 2 fingers down, counting back 1 for each finger: 10 (whole), 9, 8 – Encourage childrne to know the answer without counting (subitising). 		

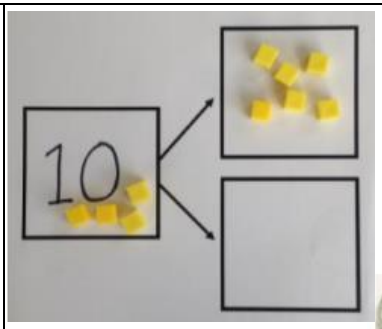
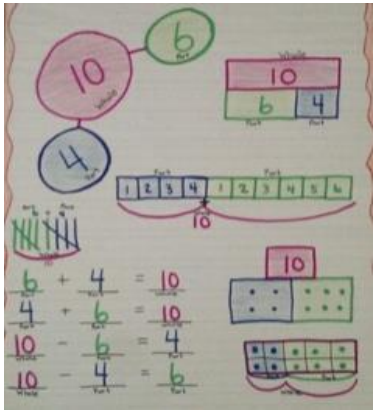
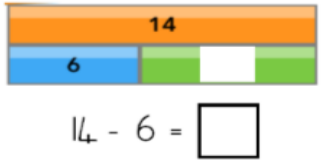
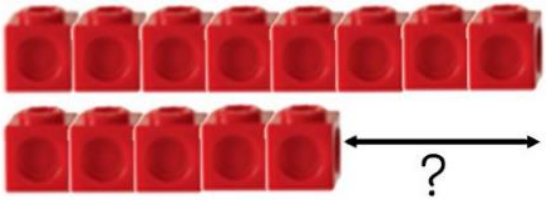
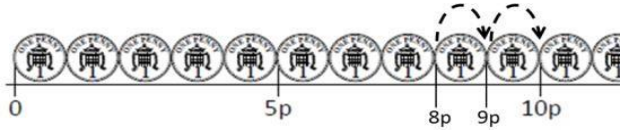
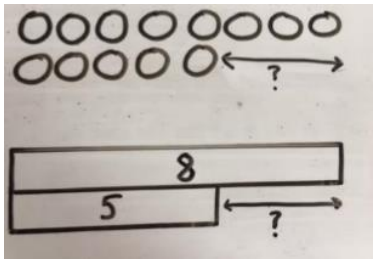
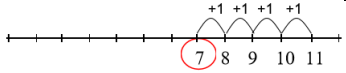
Statutory Requirements:

- Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- Represent and use number bonds and related subtraction facts within 20
- Subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems.

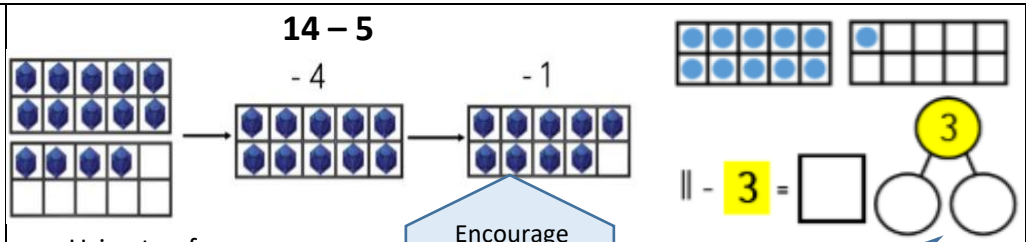
Vocabulary

take away, less than, subtract, minus, fewer, decrease, **the difference between**, **number bonds,** 'is equal to', 'is the same as', leave, how many are left/left over? how many have gone? **number line, how many more to make..?, how many more is...than..?, how much more is..? how many fewer is...than..?, how much less is..? inverse,**

Objective	Concrete and Visual representations	Imagery	Abstract
Take away using objects.	 Act out the maths story with concrete representations (objects, fingers, counters...) 7 people are on the bus. 1 is getting off at the next stop. ***How many will be left*** on the bus then? 9 – 4 = 5 6 – 2 = 4 8 – 3 = 5 7 – 5 = 2	Draw the representation of a subtraction, cross out the correct amount. E.g. 4-3 	– Recall subtraction facts within 10. – Progress to facts up to 20.
Count back to subtract small numbers.	 10 10 – 1 = 9 9 – 1 = 8  – Revise counting backwards from 10. – Progress to count backwards from 20 and beyond using marked number track or number line.	Draw a blank number line to work out a subtraction question, label the jumps correctly. 	– Recall 1 less, 2 less, 3 less than .... (any 2digit number)
See how subtraction 'undoes' addition	 Show 5 beads on a bead bar. Count on 2 more. What number sentence can we write? **5 + 2 = □** Slide the 2 beads back. What do you notice? We're back where we started! **□ – 2 = 5** **13 + 2 = 15**  **15 – 2** 	Show what this will look like on a number line. 	**Missing numbers** **7 - 3 = □** **□ = 7 - 3** **7 - □ = 4** **4 = 7 - □** **□ - 3 = 4** **4 = □ - 3** **□ - □ = 4** **4 = □ - □**

Represent and use number bonds and related subtraction facts within 10 Part – Whole model. (Teach + and – together)	 **10 – 6 = 4** **4 = 10 – 6** **10 – 4 = 6** **6 = 10 – 4** **4 + 6 = 10** **10 = 4 + 6** **6 + 4 = 10** **10 = 6 + 4**	– Model how to use part whole model, starting with the **whole** amount, one **part** being taken away, how many left is **part** 2? – Similar arrangement of 2 colours of items e.g. in egg box 10-frame, cubes or with Numicon. – Write add and subtract number sentences for each set. Vary the way = sign being placed.  	Children to draw the concrete resources they use. The bar model can also be used. 	– Recall subtraction facts within 10. – Progress to facts up to 20. – Missing number problems  – Missing operation problem. *Which sign can be used in the blanks?* E.g. 4 □ 6 □ 10
Finding the difference by counting on Find change by counting on	 - Finding change from 10p after spending 8p in the shop. - It's crucial that children understand the value of the coins and knowing that **1 ten is the same as 10 ones**. - Demonstrate using the money line and count on from 8p (doing 2 hops). 	Calculate the difference between 8 and 5, using cubes, Numicon or Cuisenaire rods, other objects	 Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.	 – Find the difference between 7 and 11 using a number line. Recording by - drawing jumps on prepared lines - constructing own lines, if appropriate – Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

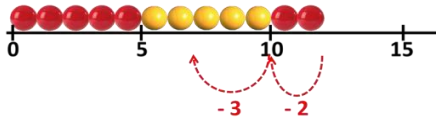
<table>
<tr>
<td>Subtracting 1 digit number from a teen number, bridging ten

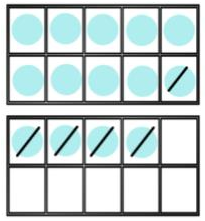
TO - O</td>
<td>
14 – 5


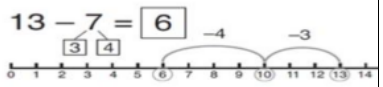
- Using ten frames, making 10 first.
- Then take away the rest.

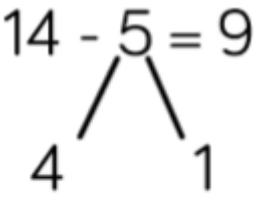
Encourage children to use their knowledge of number bonds to 10, rather than counting.

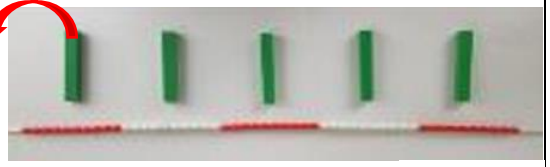
Use the part- whole model to partition the smaller number.

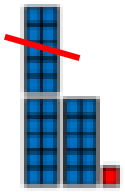
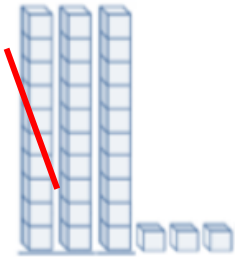
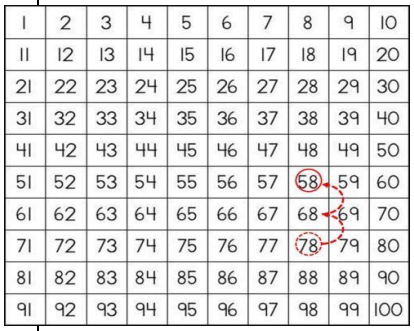
12 – 5
Using bead bar or number track.


</td>
<td>Children to present the ten frame pictorially and discuss what they did to make 10.


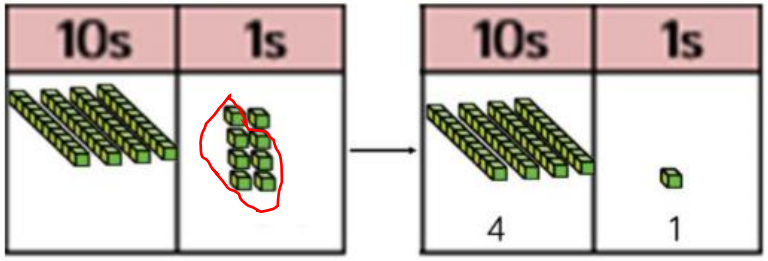
Children draw a number line, record the jumps correctly, making ten first.


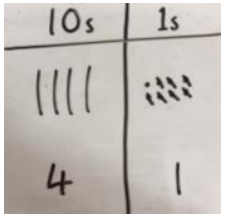
</td>
<td>


14 – 4 = 10
10 – 1 = 9
Children to show how they can make 10 by partitioning the subtrahend.
</td>
</tr>
<tr>
<td>Find 1 less and 10 less than any 2 digit number. (progress to subtracting 10s)</td>
<td>
- Count back in 1s from 100, seeing numbers on a number line and on a hundred squares. Recognise that 1 less is the number comes before the said number.
- Find 10 less using base 10. Physically remove 1 ten. “There are 5 tens, which is 50”, take 1 ten away, “10 less than 50 is 4 tens which is 40”


5 tens – 1 ten = 4 tens
50 -10 = 40
Repeat with take away 2 tens.
Recognise the pattern in the number facts. E.g. 33 -10, 31 - 10

</td>
<td>33 – 10=

Children draw the subtraction using representation of the number, cross out the correct amount of tens.</td>
<td>“Spider” moves up and down. Record the subtraction. 78 – 20 = 58.


</td>
</tr>
<tr>
<td>Formal method
Subtracting ones from a 2-digit number

TO - O
(no exchanging)</td>
<td>


48 – 7
</td>
<td>Children draw 48 using base 10. Then cross out 7.


</td>
<td>Using known number fact
8 – 7 = 1
there fore 48 – 7 = 41

generate more facts from known facts.</td>
</tr>
</table>

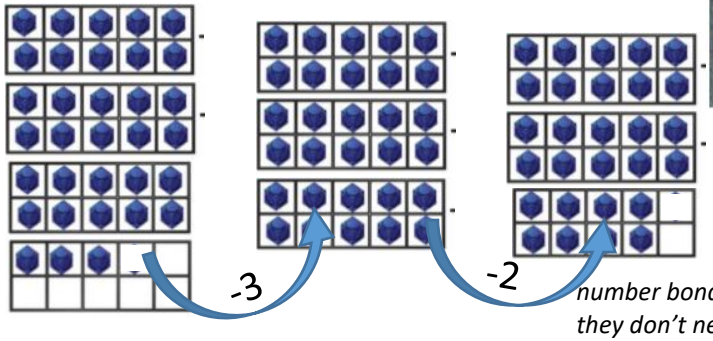
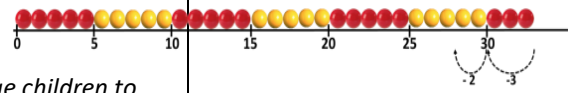
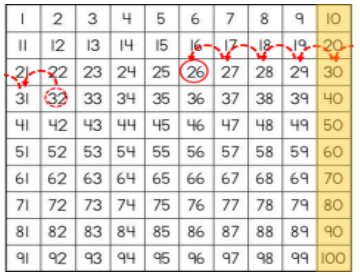
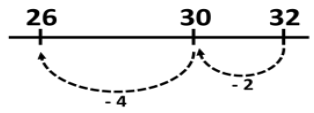
Statutory Requirements:

- Solve problems with subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures and apply their increasing knowledge of mental and written methods
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones,
  - a two digit number and tens,
  - two two-digit numbers and
  - subtract three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Vocabulary

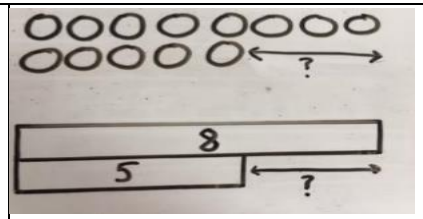
take away, less than, subtract, minus, fewer, decrease, the difference between, number bonds, is equal to, 'is the same as', leave, how many are left? how many have gone? how many more to make..?, how many more is... than ..?, how much more is..? how many fewer is...than..?, how much less is..? inverse, partition, recombine, hundred

Children build on the learning in Y1 by initially using a number line to take away and then progress to using a number line to show the difference with larger numbers; including crossing tens boundaries.

Objective	Concrete and Visual representations	Imagery	Abstract
Mental strategy Subtract a single digit from a 2-digit number by bridging multiples of ten. **TO - O**	**33 – 5 =** Take away 5 in 2 steps. Make a multiple of 10 first.    *Encourage children to use knowledge of number bonds for each step, so that they don't need to count.*	– Draw the 2 steps on a bead line to work out the subtraction. Label the jumps correctly as shown here.	***32 - 6***  

Mental strategy

Subtract by counting on. Finding the difference.



27	
15	?

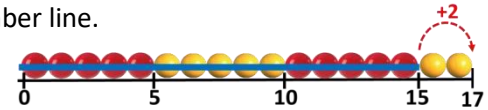
12	15
?	

?	
15	?

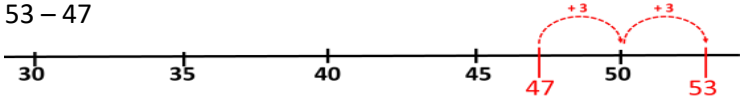
Revisit the concept of finding the difference from Y1. Using cubes and drawing of the bar model to show relationship between + and – (using words 'whole/parts'). Include spatial reasoning estimates.

When the difference is small, count on to find the difference on a bead line or number line.

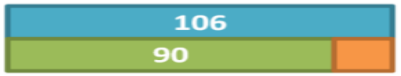
17 – 15 = 2



53 – 47



106 – 90 =

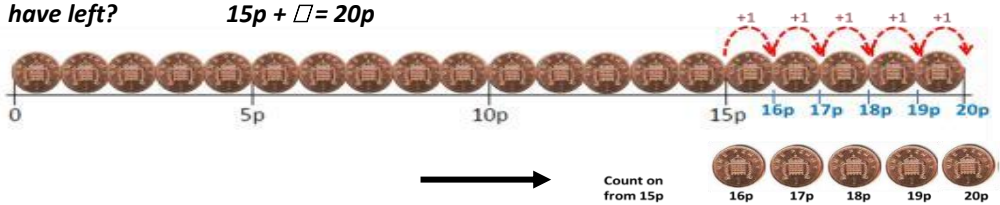


Count on from 90, in 10s then in 1s.

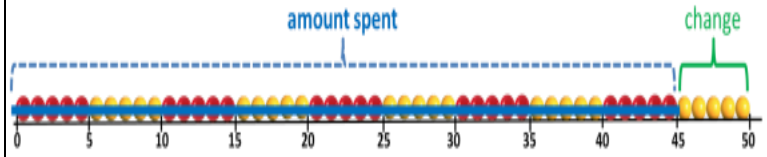
Mental strategy

Find change by counting on

Laura has 20p. She spends 15p on an apple in the school tuck shop. How much does she have left? ***15p + □ = 20p***



– Find change from 50p



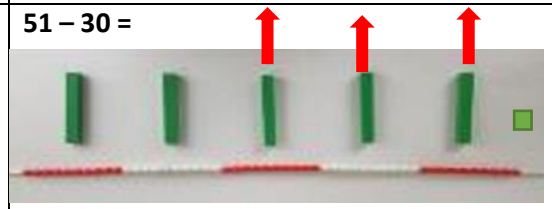
Mental strategy

Subtract 20, 30, 40, 50 from two-digit numbers

TO – 10s

Subtract near multiples of 10.

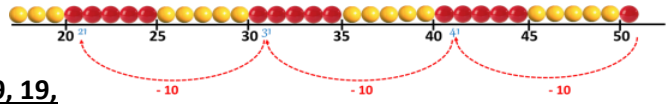
51 – 30 =



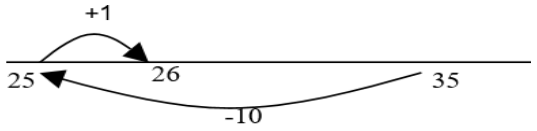
Use dienes and beads string, move 3 tens.

Observe that the one digit is not changing, only the ten digit changes.

Practice counting in 10s from any 2digit number using a beaded or landmarked number line.



<u>Subtracting 11, 21, 31, or 9, 19, 29 from two-digit numbers by subtracting 10s and adjust.</u> **35 - 9**



21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

37 – 12

53- 11

Use 100 squares, move like a spider up to subtract 10s.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

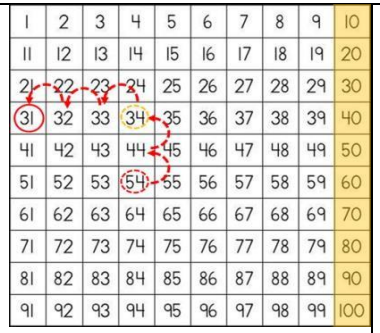
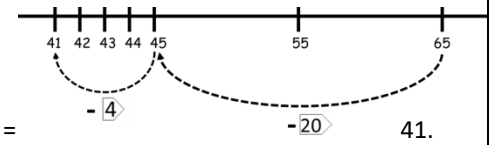
Recognising the ten digit and the one digit pattern as counting.

Progress to be able to recite counting in 10s from any digit number.

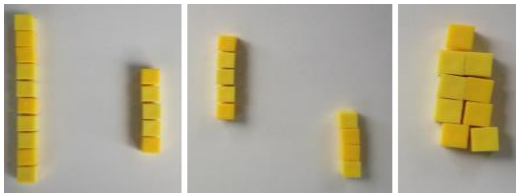
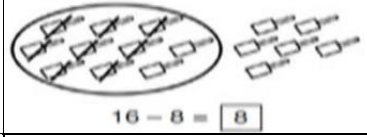
Put number 78 in their head and count back 20, in tens.

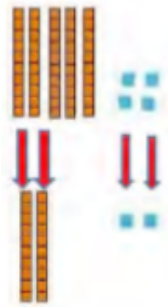
<table>
<tr>
<td>Mental strategy
Subtract 2 digit numbers by counting back.

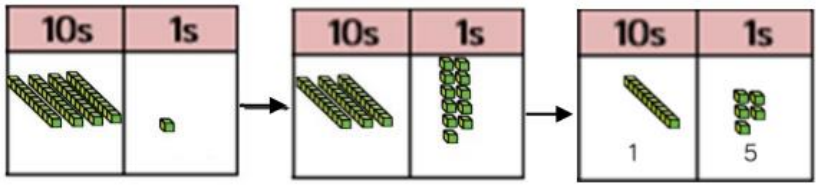
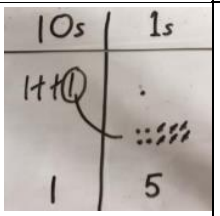
TO - TO</td>
<td>Revise counting backwards from 100. 54 – 23
1. Partition 23.
2. Draw out that we can count back 20, and then subtract 3.
3. Count back 20 in 10s from 54 (spider): 44, 34
4. Count back 3 in 1s from 34 (frog): 33, 32, 31
5. Record 54 – 23 = 31

 
</td>
<td>65 – 24
1. Partition 24 = 20 and 4
2. Use a blank number line, count back 20 from 65 (all in one go or in 2 jumps of 10), label landing on 45. Then count back 4 more. We know 5 - 4 is one, so 45 - 4 is 41. So we still don't need to count back in ones!
3. Write the answer to complete the subtraction. 65 – 24 = 41.
 
</td>
</tr>
<tr>
<td>(preparing for formal method)
Subtract 1 digit number from a multiple of 10

TO - O</td>
<td> 
20 – 4
exchanging 1 ten to ten ones, then take 4 ones away

 
15 – 6
Pupils should identify that they can also take away from the tens and get the same answer. This reinforces their knowledge of NB to 10 and develops their mental strategies.</td>
<td> 
– Exchange 1 stick (ten) with 10 ones.
– Draw and cross out 4 ones after exchanging.
 
</td>
<td>20 – 4 =
10 + 10 – 4 =
10 + 6 = 16

15 – 6 = 10 + 5 – 6
= 4 + 5 = 9</td>
</tr>
<tr>
<td>Subtract 2 digit number from a 2 digit number.
TO – TO
<u>Without exchanging</u></td>
<td>54 – 22
Use Dienes to represent number 54. Then take away 2 tens and 2 ones (make 22). Count what's left.
  </td>
<td>Draw tens and ones on a place value chart. Cross out 1 ten and 3 ones (make 13) and count what's left.
34 – 13
  </td>
<td>34 – 13
30 + 4
- 10 + 3
20 + 1 = 21</td>
</tr>
<tr>
<td>Subtract 2 digit number.

TO – TO
<u>With exchanging</u></td>
<td>41 - 26
 
</td>
<td>Represent the base 10 pictorially, show the exchange clearly.
 

Then cross out 2 tens, 6 ones</td>
<td>40 + 1 → 30 + 11
- 20 + 6 → 20 + 6
= 10 + 5</td>
</tr>
</table>

YEAR 3 **SUBTRACTION**

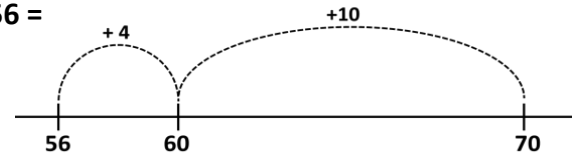
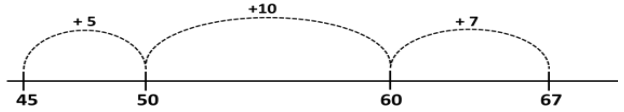
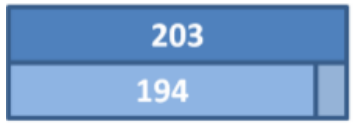
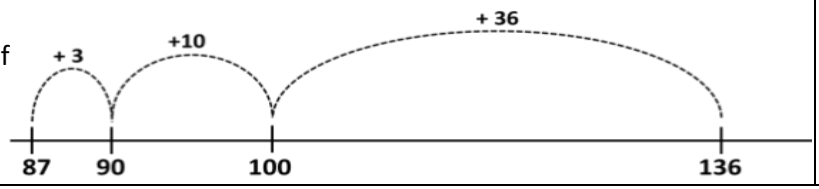
Statutory Requirements:

- Subtract numbers mentally, including:
  - a three-digit number and ones,
  - a three-digit number and tens,
  - a three-digit number and hundreds,
  - a three-digit number and thousands
- Subtract numbers with up to three digits, using formal written methods of column subtraction where appropriate
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

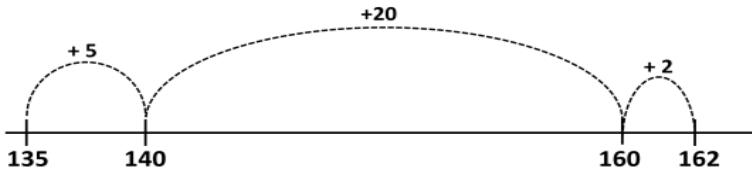
Vocabulary

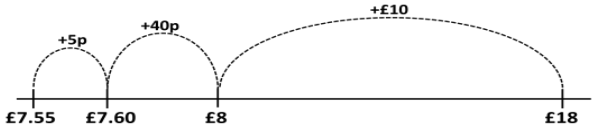
take away, less than, subtract, minus, fewer, decrease, the difference between, number bonds, is equal to, 'is the same as', leave, how many have gone? how many more to make..?, how many more is... than ..?, how much more is..? how many fewer is...than..?, how much less is..? inverse, partition, recombine, hundred, column method

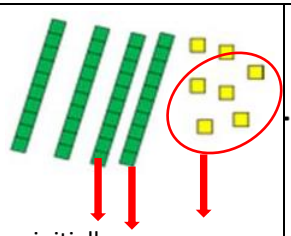
It is essential that this builds on previous learning and their knowledge and understanding of place value. Formal method should not replace effective mental strategies and at all stages children should be encouraged to use their number sense to decide on the most appropriate methods.

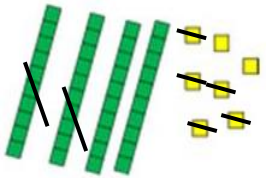
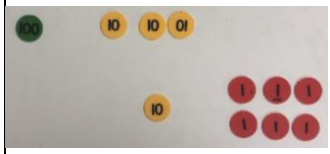
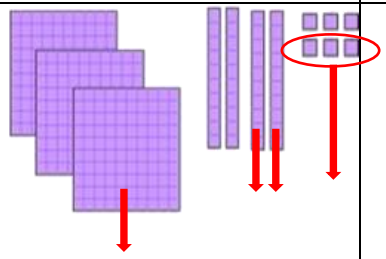
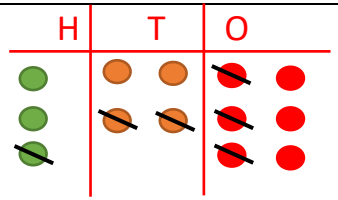
Objective	Concrete and Visual representations	Imagery	Abstract
Mental strategy Subtract 2 digit number by counting up. **TO - TO**	**Subtract by counting up (answers less than 20)** **70 – 56 =**  Jump from smaller number to the next multiple of ten first, then to the bigger number. Add up all the jumps: 10 + 4 = 14 so 70 – 56 = 14 Extend to subtracting by counting up to include numbers on **either side of 100.** **106 – 90** = 10 + 6 = **16**	**Subtract by counting up (answers more than 20)** **67 – 45** = 5 + 10 + 7 **= 22**  **106 – 90 =**  	Count up in their head without using a visual support. **203 - 194**  Mentally work out the difference: 6 + 3= 9
HTO - TO	**136 – 87 =** Jump from smaller number to the next multiple of ten first, then to 100, then the bigger number. Add up all the jumps: 3 +10 + 36 = 49 So **136 – 87 = 49**  		

<table>
<tr>
<td>HTO – HTO

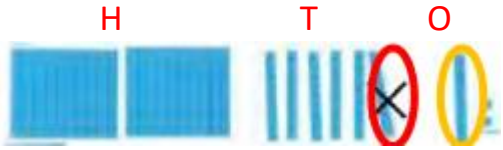
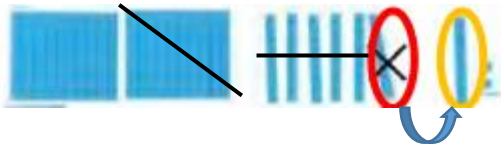
Find a difference between pairs of numbers within the same century</td>
<td>162 – 135 = 5 + 20 + 2 = 27
 

Jump from the smaller amount to the next multiple of 10p, then to the next pound, finally to the bigger amount.
Add up all the jumps. 5p + 40p + £10 = £10.45</td>
<td><u>Count up to find the difference between amounts of money</u>
A computer game costs £18. So far Katie has saved up £7.55. How much more does she need to save to be able to buy the game?
 

£18 - £7.55 = £10.45</td>
<td></td>
</tr>
<tr>
<td rowspan="2">Formal method
Expanded Column subtraction (without any exchanging)</td>
<td>47 – 25
Dienes can be replaced by place value counters. Physically move the tens and ones to take away the correct amount.
 
* Modelling practical alongside formal written initially.
* Move to formal columnar strategy using labelled columns and starting with numbers not requiring exchange before strategy and understanding is secure.</td>
<td>T | U
40 | 7
20 | 5
20 | 2 = 22

Children draw represent the base 10 pictorially. Cross out the correct amount of tens and ones to subtract 25.</td>
<td>Use place value cards to help partition and use a place value grid to help record.
  </td>
<td>T | O
40 | 7
- 20 | 5
20 | 2 = 22</td>
</tr>
<tr>
<td>346 – 123 =
Dienes can be replaced by place value counters.
 
(146 – 16)
 
* When/if children are secure with this then they can be exposed to exchanging using concrete resources initially.
Start with single exchanges.</td>
<td>H | T | O
 
Children draw represent the base 10 pictorially. Cross out the correct amount

H | T | U
300 | 40 | 6
– 100 | 20 | 3
200 | 20 | 3 = 223

Use place value cards to help partition and use a place value grid to help record.</td>
<td>H | T | O
300 | 40 | 6
- 100 | 20 | 3
200 | 20 | 3 = 223</td>
</tr>
<tr>
<td>Expanded Column subtraction (with a single exchange)</td>
<td>262 – 154
 
</td>
<td>Draw Dienes. Exchange 1 ten to 10 ones.
  </td>
<td>H | T | O
200 | ~~60~~ 50 | 12
- 100 | 50 | 4
100 | 0 | 8 = 108</td>
</tr>
</table>

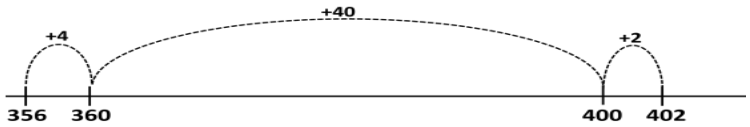
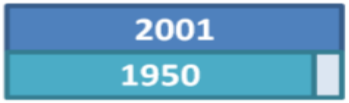
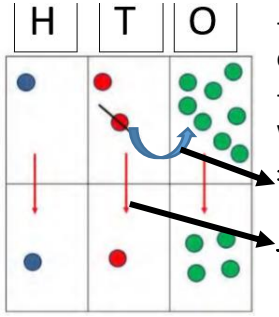
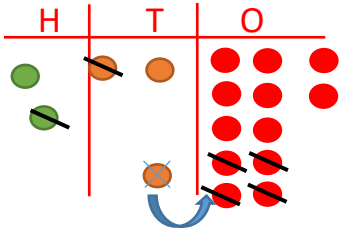
Statutory Requirements:

- Subtract with up to 4 digits using the formal written methods of column subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

Vocabulary

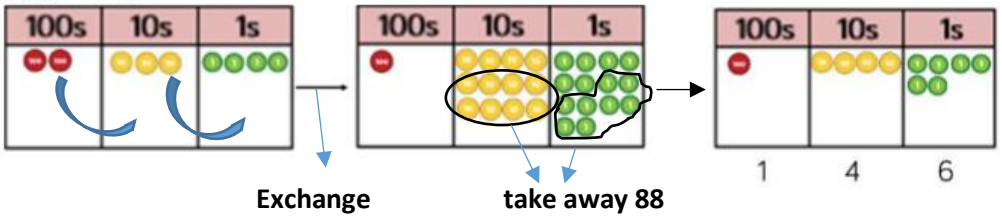
take away, less than, subtract, minus, fewer, decrease, the difference between, inverse, partition, recombine, hundred, is equal to, 'is the same as', leave, how many have gone? how many more to make..?, how many/much more is... than ..?, how many fewer/much less is... than..?, number bonds, column method, thousand more/less, expanded, compact, estimate, efficient

It is essential that this builds on previous learning and their knowledge and understanding of place value. The methods taught in Y3 should be the starting point and concrete resources and pictorial representations remain essential. Numbers will increase in size and the children will be exposed to more exchanging. Formal method should not replace effective mental strategies and at all stages children should be encouraged to use their number sense to decide on the most appropriate methods and use rounding to estimate answers.

Objective	Concrete and Visual representations	Imagery	Abstract
Mental strategy Subtract 3 digit number by counting up. **HTO - HTO**	**<u>Review on Mental strategy from Y3 before progress to:</u>** Find a difference between pairs of numbers **with different century** **402 – 356** =  	Jump from **smaller number** to the **next multiple of ten first**, then to the **next hundred**, finally to the **greater number**. Add up all the jumps: 40 + 4 + 2 = 46 so 402 – 356 = 46	*Count up or down in their head without using a visual support.* *503 – 15* *304 - 199* **2001 - 1950** 
Formal method Expanded and compact subtraction of 3 digit numbers. <u>With a single exchange</u>	**232 – 114**  - Model writing the formal method alongside the practical example. - Take care with choice of number initially. Avoid examples where a mental strategy is more appropriate. **When/if children are ready, progress to example of subtraction of 4 digit numbers with one exchange.**	Drawing of base ten imagery, indicating the exchange and cross out the correct amount.  * estimate by rounding to check the answer.	200 30 2 - 100 10 4 100 10 8 ***Progress to compact recording*** H T O 2 3 2 - 1 1 4 1 1 8 Th H T U 8 12 5 9 2 7 - 3 4 5 6 2 4 7 1

Formal method
Expanded/ Compact subtraction of 3 digit numbers.
With more than one exchange.

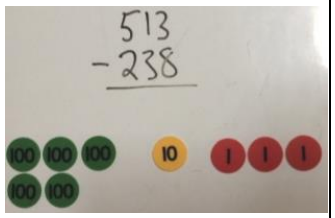
234 - 88



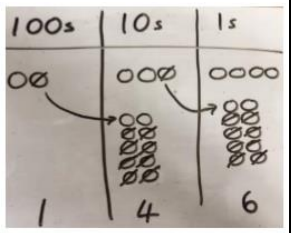
Progress to examples of HTO – HTO with 2 exchanges, following the same steps. Model writing the formal method alongside the practical example.

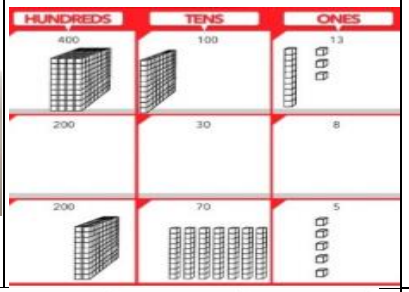
E.g. 513 – 238

* estimate by rounding to check the answer.



234 – 88





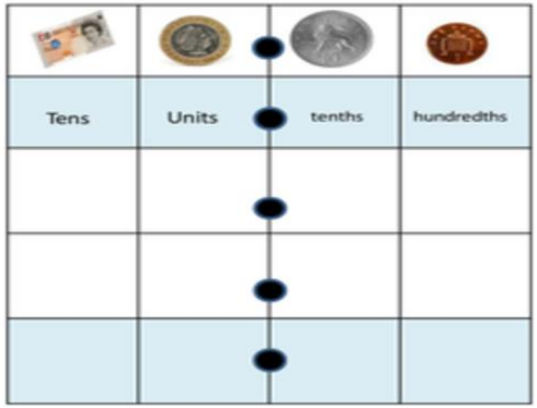
H T O
2 1
2 3 4
- 8 8
1 4 6

H T O
4 10 1 13
5 1 3
- 2 3 8
2 7 5

Expanded column subtraction of decimal numbers in money context.
(without exchanging)

I have £86.75 in the bank and a pair of shoes costs £35.42. Can I afford it? How much money will I have left if I buy the shoes?

* Be mindful of number choice here! Whenever possible, children should be encouraged to use mental strategy and their knowledge of number facts before carrying out formal (column method)



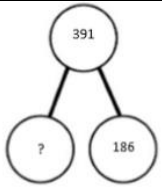
£10	£1	10p	1p
80	6	70	5
− 30	5	40	2
50	1	30	3

= £51.33

* estimate by rounding to check the answer.

Applying

Conceptual Variation



391	
186	?

□ = 391 - 186

391
-186
___

What is 186 less than 391?

Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.

missing digit problem

3 9 □
- □ □ 6
□ 0 5

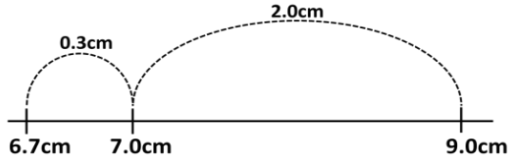
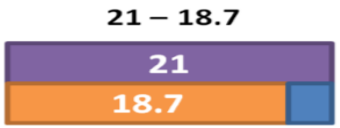
Statutory Requirements:

- subtract whole numbers with more than 4 digits, including the use of column subtraction
- Subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

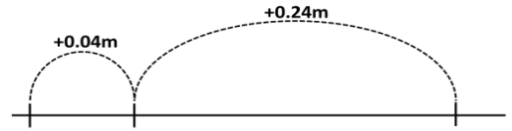
Vocabulary

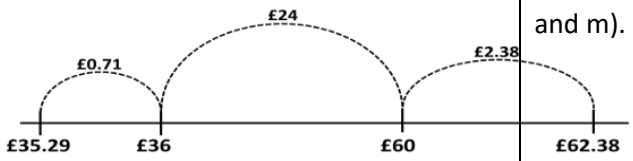
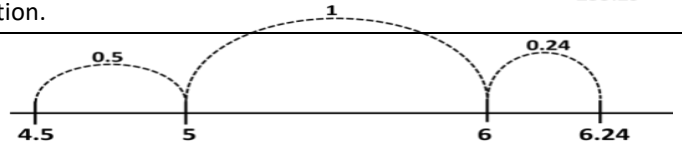
take away, less than, subtract, minus, fewer, decrease, the difference between, number bonds, is equal to, 'is the same as', leave, how many have gone? how many more to make..?, how many more is... than ..?, how much more is..? how many fewer is...than..?, how much less is..? inverse, partition, recombine, hundred, column method, thousand more/less, expanded, compact

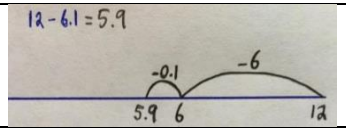
Subtraction in Y5, whole numbers will increase in size, decimal numbers with different decimal places and the children will be exposed to more exchanging. Formal method should not replace effective mental strategies and at all stages children should be given the opportunity to use their number sense to decide on the most appropriate methods and use rounding to estimate answers.

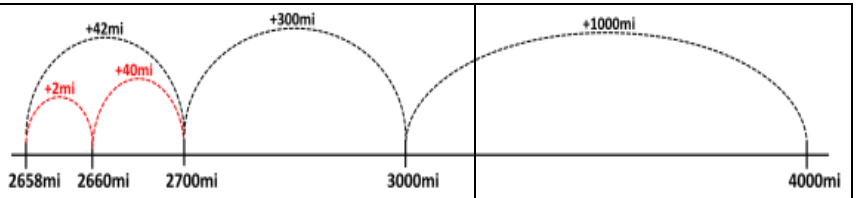
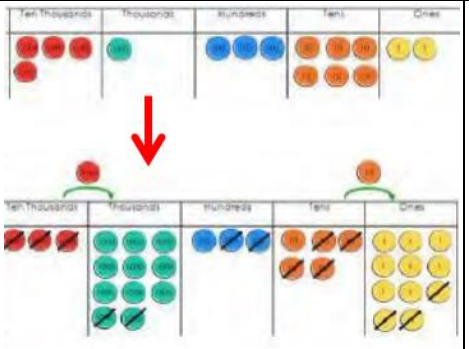
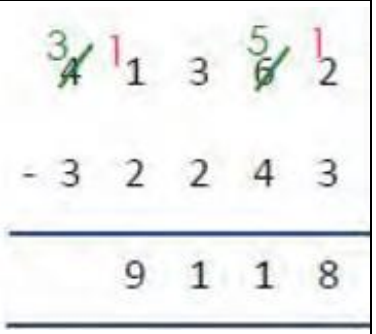
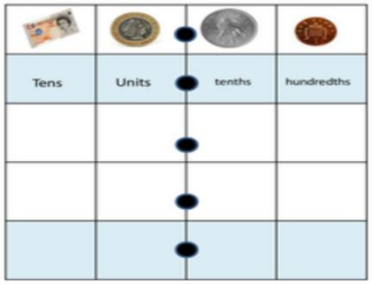
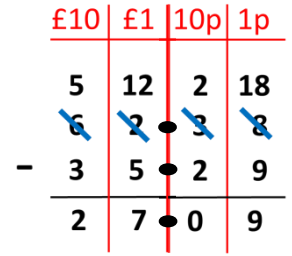
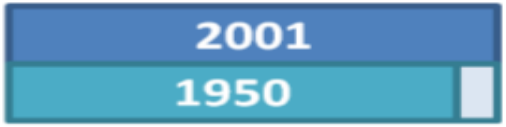
<table>
<tr><th>Objective</th><th>Concrete and Visual representations</th><th>Imagery</th><th>Abstract</th></tr>
<tr><td>Mental strategy
Subtract pairs of numbers with 1 decimal place by counting up.</td><td>The average length of a mouse is 9cm. Say that a young mouse is 6.7cm long.
How much more is it likely to grow?
1. Jump from the smaller number to the nearest whole number. Draw a hop labelled 0.3cm (use bonds to 10!),
2. Then a jump to the greater number. Draw and label the jump.
3. Add up the jumps on the number line to find the answer. 2.3cm.</td><td> 
</td><td>Count up in their head.
 
</td></tr>
<tr><td>Subtract pairs of numbers with 2 decimal places by counting up.

Subtract pairs of numbers with different numbers of decimal places (1 or 2)</td><td>Cindy's best long jump this year was 2.96 metres, but today she has jumped a huge 3.24 metres! How much further has she jumped?
1. Jump from the smaller number to the nearest whole number. Draw a hop labelled the difference (use bonds to 100).
2. Then a jump to the greater number. Draw and label the jump.
3. Add up the jumps on the number line to find the answer.

£62.38 - £35.29
Jump to the nearest whole number. The number of jumps depends on children's knowledge of mental calculation.</td><td> 

 
</td><td>* It is important that the children are secure in their knowledge of the decimal place value and knowledge of measuring units (e.g. cm and m).</td></tr>
<tr><td></td><td>6.24 – 4.5 =
 

 
</td><td></td><td></td></tr>
</table>

<table>
<tr>
<td>Mental strategy
Subtract 4 digit number from a multiple of 1000.</td>
<td>A group of people are cycling 4000 miles. So far they have travelled 2658 miles, so over half way. How much further have they got to go?
1. Using an empty number line, hop from 2658 to the nearest 10 (which is 2660), then to the nearest 100 (which is 2700), then to the nearest 1000 (3000) then to the final greater number (4000). Label each jump.
The number of hops (jumps) depends on children's knowledge of number bonds to 10, 100, 1000.
2. Add the hops and jumps to find out how much further the cyclists have to go.</td>
<td> 
</td>
</tr>
<tr>
<td>Formal method
Use column subtraction (decomposition) to subtract pairs up to five-digit numbers</td>
<td>* Use of place value counters and PV grid to scaffold the learning if needed.
* Use of PV cards to support partitioning and exchanging to scaffold the learning if needed.
* Write as an expanded vertical subtraction where support is needed.
* Estimate the answer by rounding. (E: 41000 – 32000 = 9000)</td>
<td>  </td>
<td> 
</td>
</tr>
<tr>
<td>Formal method
Use column subtraction (decomposition) to subtract pairs of decimal numbers.</td>
<td>* Use of money or place value counters to scaffold the learning if needed.
* Write as an expanded vertical subtraction where support is needed.
* Estimate the answer by rounding. (E: £62 – £35 = £27)</td>
<td>£62.38 - £35.29
 
</td>
<td> 
</td>
</tr>
<tr>
<td>Applying</td>
<td>- Decide on the most effective method. Give children a variety of calculation and the children give reasons to which method they would use.
E.g. 30,001 – 29,999 = ?
24,220 – 1120 = ?
- Solve measuring and money problems using + and – skills
- Solve missing number problems, noting the missing quantities as symbols x, y, z.</td>
<td colspan="2">2001 - 1950
 
</td>
</tr>
</table>

Statutory Requirements:

- Perform mental calculations, including with mixed operations and large numbers
- Use my knowledge of the order of operations to carry out calculations involving the 4 operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

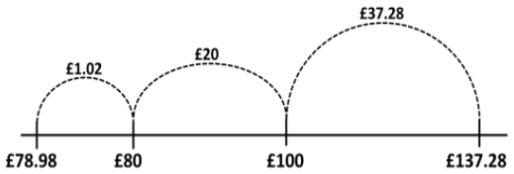
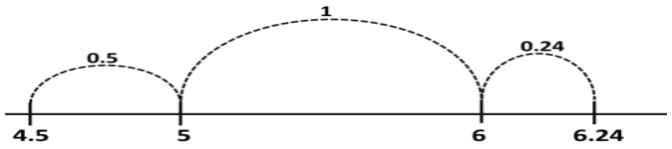
Vocabulary

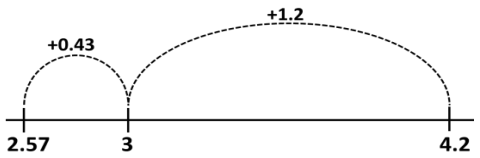
take away, less than, subtract, minus, fewer, decrease, the difference between, number bonds, is equal to, leave, inverse, partition, recombine, **ten/hundred thousand**, thousand, hundred, column method, thousand more/less, expanded, compact, **order of operations.**

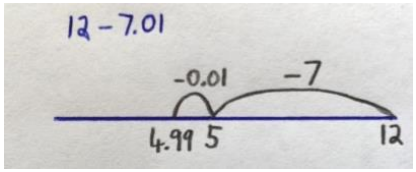
In Y6, children will be consolidating and building on existing strategies that they have been taught so far, with numbers increase in size (up to 10,000, 000), and more complex decimal numbers (up to 3 decimal places) and the children will be exposed to more exchanging.
Formal method should not replace effective mental strategies and at all stages children should be given the opportunity to use their number sense to decide on the most appropriate methods and use rounding to estimate answers.

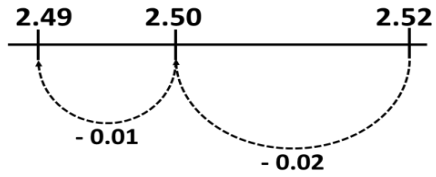
<table>
<tr><th>Objective</th><th>Concrete and Visual representations</th><th>Imagery</th><th>Abstract</th></tr>
<tr><td rowspan="3">Mental strategy
Subtract multiples of 0.01 from numbers with 2 decimal place, crossing multiples of 0.1</td><td colspan="2">Begin with 12 – 7.01
Children should be fluent in counting on/back in 0.1, 0.01 , 0.001 cross boundaries.
Progress to 2.52 – 0.03
1. Count back from 2.52 to 2.49 to find the answer.
2. Sketch a jotting on the board to show how we can ‘bridge’ 2.5.</td><td rowspan="3">Count on/back in their head, knowing the nearest boundary.
Understand the decimal place value.

Write 34.567
1. If we add 0.1, which digit will change? If we subtract 0.01 which digit will change? Which digit will change if we add or subtract 0.001?
2. Repeat, adding and subtracting multiples of 10, 1, 0.1 and 0.01, and 0.001, checking that chln changing the correct digit each time. (Make sure that the + or - do not cross through a multiple of 0.1 or 1 boundary, e.g. not 34.67 + 0.05 or 34.23 – 0.4.)
3. Write 34.567 again. Subtract 0.008 by counting back in steps of 0.001 taking care when crossing 34.56.</td></tr>
<tr><td rowspan="2">Subtract pairs of numbers from different century or/and with different numbers of decimal places (1 or 2)</td><td>£137.28 - £78.98
I went shopping and to start with I had £137.28 in my bank account. I spent £78.98. How much did I have left?

Jump to the nearest whole number. The number of jumps depends on children’s knowledge of mental calculation.</td><td> 
</td></tr>
<tr><td colspan="2">6.24 – 4.5 =
 

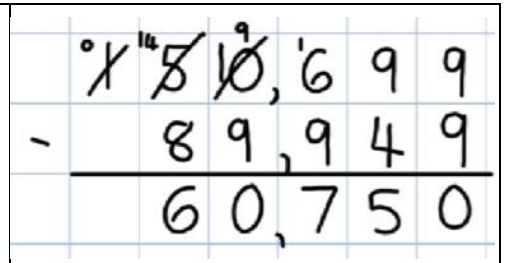
4.2 – 2.57 =
 
</td></tr>
</table>



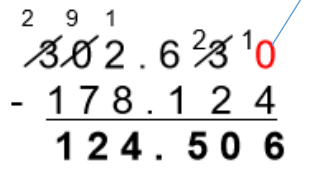


<table>
<tr>
<td>Formal Method

Compact subtraction with up to 6 digit numbers

<u>(more than one exchanges)</u></td>
<td>Use of concrete resources to support (e.g. place value) if needed
 
</td>
<td> 

Estimate: 150,000 – 90,000= 60,000</td>
</tr>
<tr>
<td>Compact subtraction with pairs of numbers with different decimal places.

<u>(more than one exchanges)</u></td>
<td>  </td>
<td>302.63 – 178.124 =
 

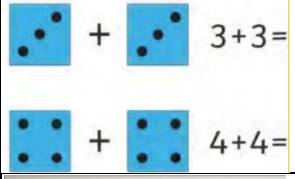
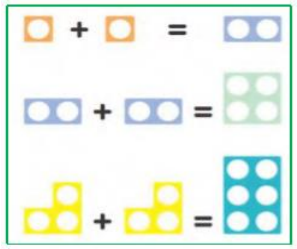
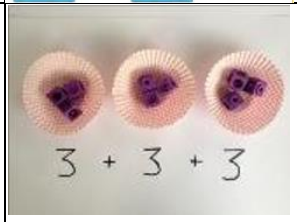
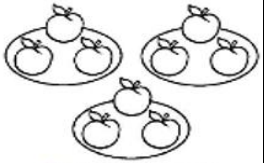
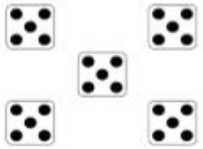
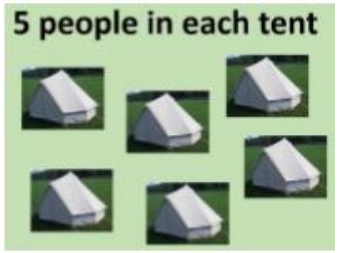
Empty decimal places can be filled with zero to show the place value in each column.</td>
</tr>
<tr>
<td>Applying</td>
<td>- Decide on the most effective method. Give children a variety of calculation and the children give reasons to which method they would use.
E.g. 30,001 – 29,999 = ?
24,220 – 1120 = ?
- Solve measuring and money problems using + and – skills
- Solve missing number problems, noting the missing quantities as symbols x, y, z.</td>
<td></td>
</tr>
</table>

Statutory Requirements:

Early Learning Goal - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Vocabulary

double, halves, the same, lots of, groups of, times (once, twice etc.), multiply, add again and again, repeated grouping, repeated adding, (how many) equal groups, total, is equal to, 'is the same as', counting in 2s, 10s , odd, even

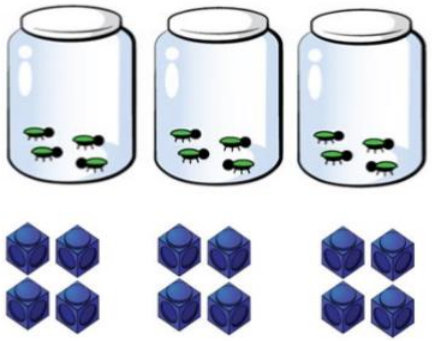
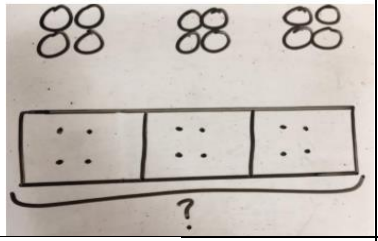
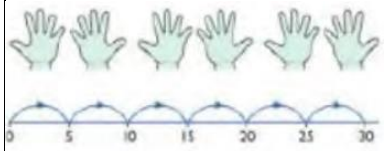
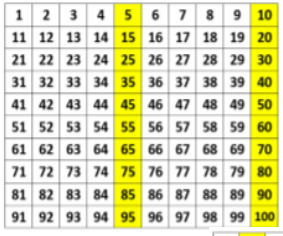
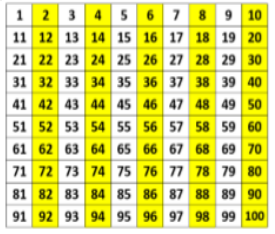
Objective	Concrete and Visual representations	Imagery → Abstract
Understand **double**	  Early understanding of multiplication will build on previous learning with addition – initial links should be introduced by **doubling** using concrete resources and pictorial representations. Halves and doubles identified in a range of contexts, with a focus on equal halves. Shown on 10-frames, cubes, beads and Numicon. ⊗ Make clear that doubling is adding the same number.  	⊗ Counting in 2s, 10s, supported by colouring of 100square  
Develop pre-multiplication concept by making equal groups and find the total by repeated adding	 ⊗ Children create equal groups using concrete resources and pictorial presentations. Then count the total. Opportunities for '**repeat add'** counting in context e.g. counting socks.    Grouping context tasks provided. How many groups of 2 are there? Count in 2s to find the total. Show me 4 equal groups of 5.  	

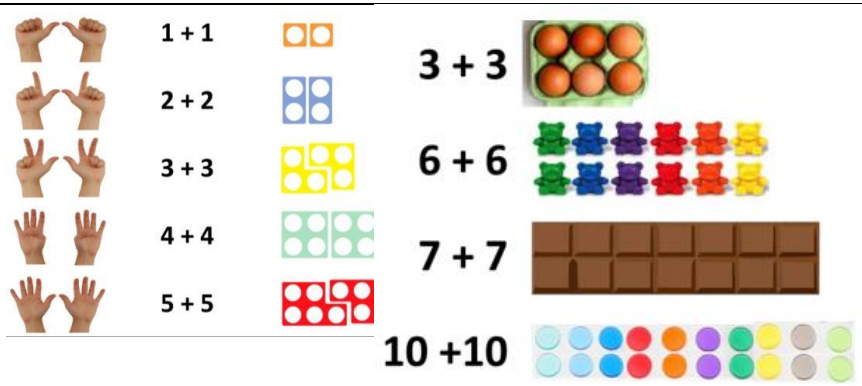
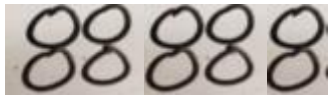
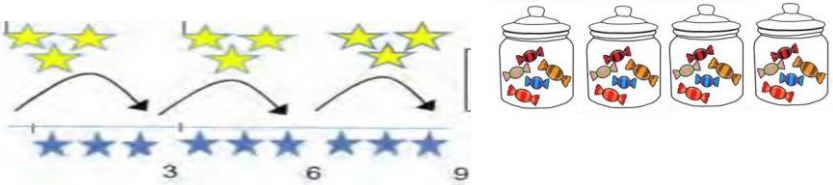
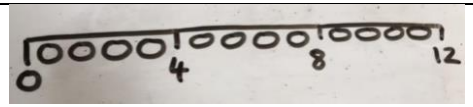
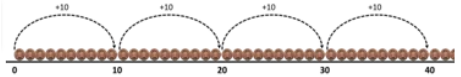
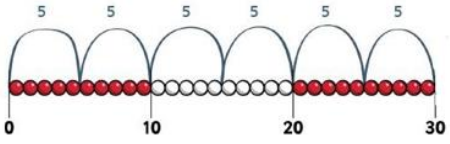
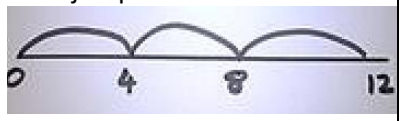
Statutory Requirements:

⊗ Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary

odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), add again and again, repeated grouping, repeated adding, (how many) equal groups, total, is equal to, 'is the same as', counting in 2s, 3s, 5s, 10s , (forwards from/backwards from), how many times? multiple of, times, multiply, multiply by, repeated addition

Objective	Concrete and Visual representations	Imagery	Abstract
Understand repeated (equal) grouping and **repeated addition**	 There are 3 equal groups, with 4 in each group. There are 3 equal groups of 4. 3 × 4 4 + 4 + 4	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Count in multiples of 2, 5, 10 and record number sentences, using X, = symbols	 2 + 2 + 2 + 2 + 2 = 10     Children initially count in groups e.g. in 2s, 5s and 10s 10, 20, 30, 40, 50	 Children can draw images of equal groups (dots, crosses or squares) to represent and calculate a given multiplication. 	⊗ Record as repeated addition initially. **2 + 2 + 2 + 2 = 8** ⊗ Progress to use X and = symbols: 4 lots/groups of 2s → **4 X 2= 8**

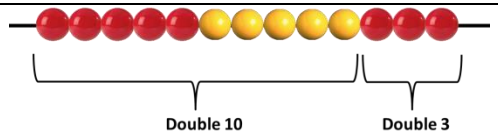
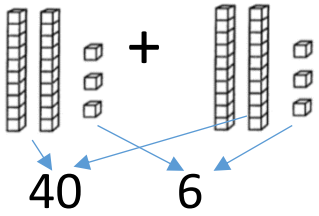
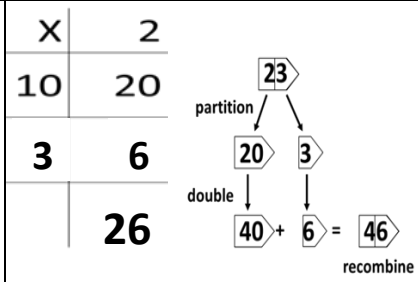
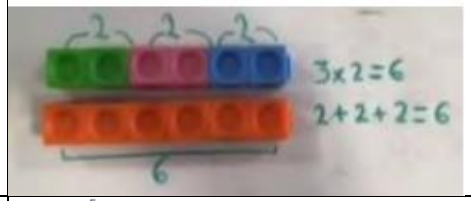
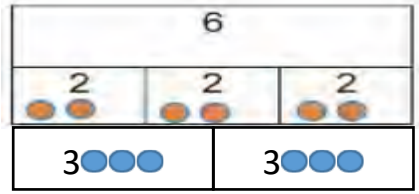
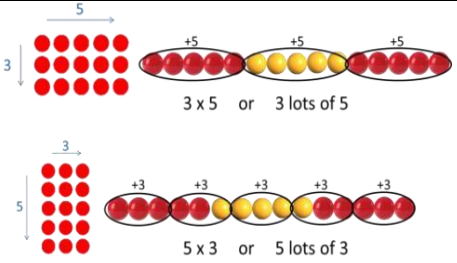
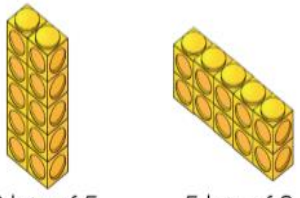
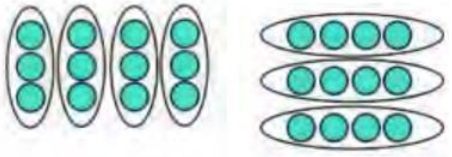
Know all double facts to 10		Draw counters in groups of 2s 	⊗ Recite double facts in order, then progress to a random recall of double facts from 1 to 10.
Use a number line to show multiplication	  Cuisenaire rods can be used too.  	 Represent this pictorially alongside a number line.  Solve money problems using penny number line. 	⊗ Blank number line to show 3 jumps of 4.   
Applying	⊗ Simple one step problems using number lines to support.		

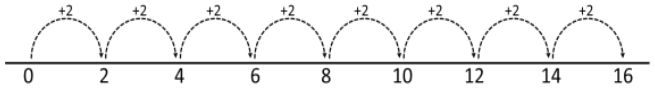
Statutory Requirements:

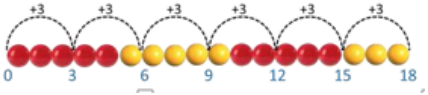
- ⊗ Recall and use multiplication facts for the 2, 3 and 5 and 10 multiplication tables, including recognising odd and even numbers
- ⊗ Calculate mathematical statements for multiplication within the 2, 5, 10 tables and write them using the multiplication (×) and equals (=) signs
- ⊗ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- ⊗ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

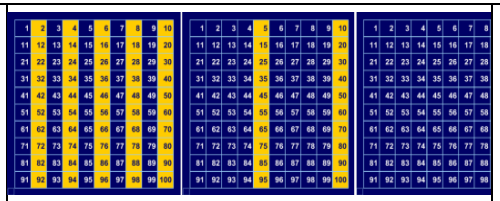
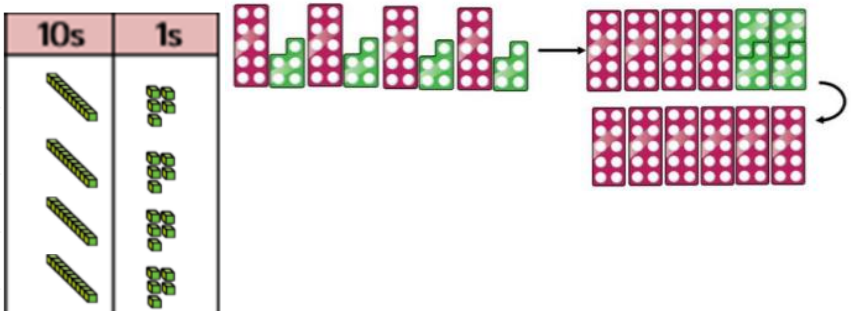
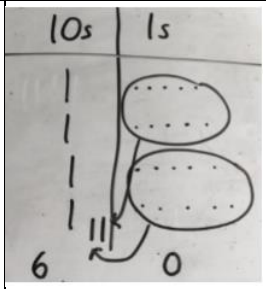
Vocabulary

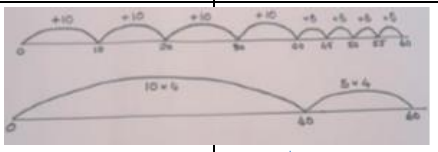
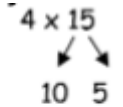
odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), add again and again, repeated grouping, repeated adding, (how many) equal groups, total, is equal to, 'is the same as', counting in 2s, 3s, 5s, 10s , (forwards from/backwards from), how many times? multiple of, multiply, multiply by, repeated addition, commutative law

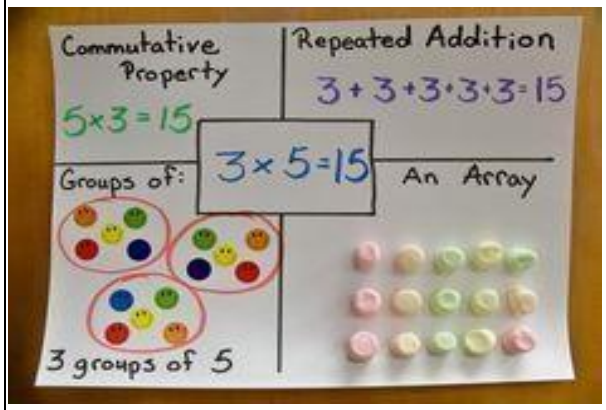
Objective	Concrete and Visual representations	Imagery	Abstract
Find doubles of 2 digit number by partitioning	***Double 13***  10 + 10 = 20 3 + 3 = 6 20 + 6 =	Children to represent the practical resources in a picture  	
Show multiplication is **commutative** using a bar model	 *It's a good idea to make 2 groups of 3 below this model to show that* ***3 X 2 = 2 X 3 = 6***	 	2 + 2 + 2 = 6 3 x 2 = 6 3 + 3 = 6 2 x 3 = 6
Show multiplication is **commutative** using arrays	  	 Children draw array using dots or squares, showing equal groups in rows and columns.	2 × 5 = 5 ×2 = 10 5 + 5 + 5 = 3 x 5 = 15 3 + 3 + 3 + 3 + 3 = 5 x 3 = 15 3 + 3 + 3 + 3 = 4 x 3 = 12 4 + 4 + 4 = 3 x 4 = 12

<table>
<tr>
<td>Multiply by 2, 5 and 10 using number lines Recognise multiples of 2, 5 and 10 and describe patterns</td>
<td>2 x 8 =
 



Count in 3s using a number line.
Progression may take place to the 4 times table by doubling and doubling again.</td>
<td> 
<table>
<tr><td colspan="4">12</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>3</td></tr>
</table></td>
<td>Fluent recall of the 2,5 and 10 times table.
1. Which of these are multiples of 5? How do you know?
2. Which of these are multiples of 10? How can you tell?</td>
</tr>
<tr>
<td>Multiply teen numbers by partitioning</td>
<td> 
</td>
<td> 

pictorially
 
Children represent the concrete manipulatives
<table>
<tr><td>X</td><td>4</td></tr>
<tr><td>10</td><td>40</td></tr>
<tr><td>5</td><td>20</td></tr>
<tr><td>=</td><td>60</td></tr>
</table></td>
<td> 

10 x 4 = 40
5 x 4 = 20
40 + 20 = 60</td>
</tr>
<tr>
<td><u>Applying Conceptual variation</u></td>
<td> 

Create poster to show understanding of multiplications.

They can add the multiplication on a number line.</td>
<td>Real life problems involve measurements, money, etc.</td>
<td>A multiplication grid with missing integers are used to reinforce the relationship between multiplication and division.
<table>
<tr><td>X</td><td></td><td>5</td><td></td></tr>
<tr><td>10</td><td>100</td><td></td><td></td></tr>
<tr><td></td><td></td><td>25</td><td></td></tr>
<tr><td>2</td><td></td><td></td><td>4</td></tr>
</table></td>
</tr>
</table>

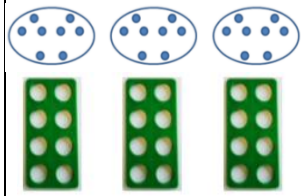
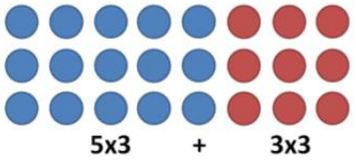
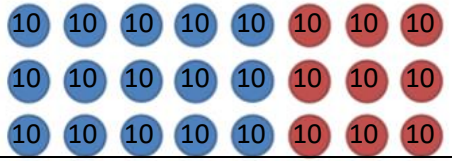
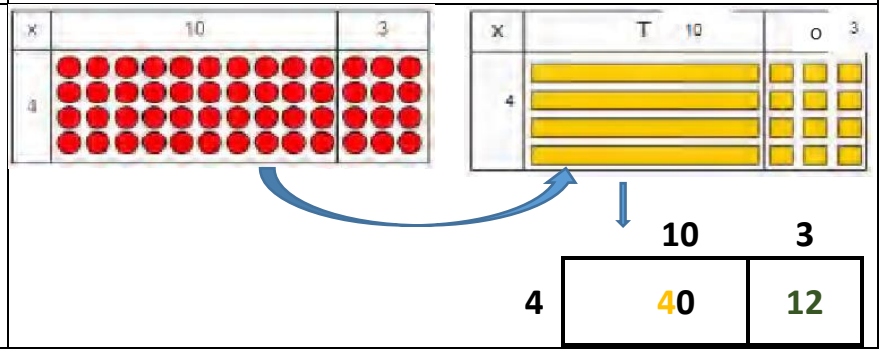
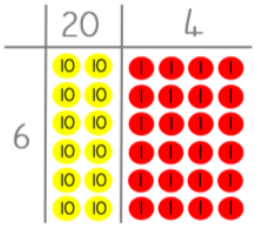
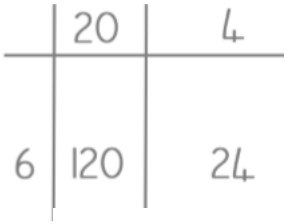
Statutory Requirements:

- ⊗ Recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- ⊗ Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental and progressing to written methods
- ⊗ Solve problems involving missing number problems involving multiplication including positive number scaling problems and correspondence problems where n objects are connected to m objects.

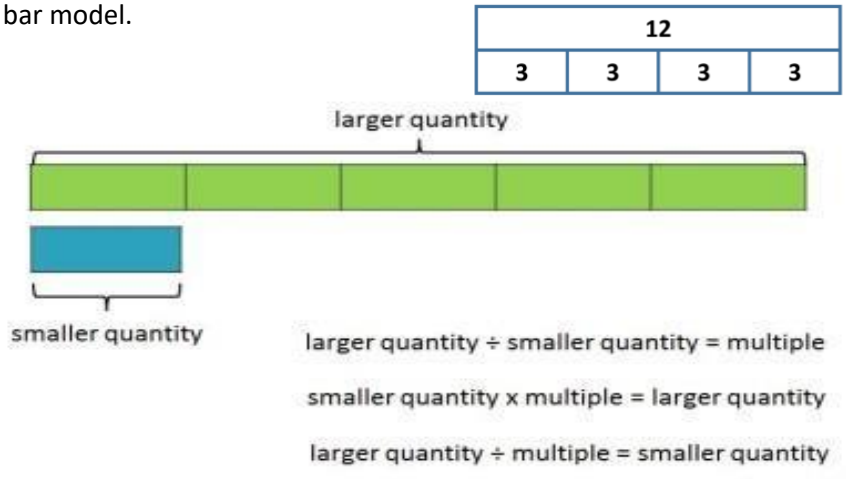
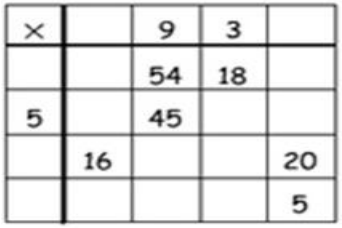
Time tables - Pupils recall x2, x5, x10, x3, x4, x6, ,x8 and x9. For x4 and x8 use doubling to help recall.

Vocabulary

odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), repeated adding, (how many) equal groups, total, is equal to, times tables, Counting in 10s, 100s, how many times? multiple of, multiply, multiply by, repeated addition, scale up, distributive law, commutative law

Objective	Concrete and Visual representations	Imagery	Abstract
Distributive law of multiplication Generate new multiplication facts using known fact	***8 x 3*** Revise on what they've learned so far.  A range of images show multiplication as repeated addition. 2-colour arrays show distributive law.  5x3 + 3x3	Children draw image of the apparatus they use. They might label each counter with different value, such as 1, 10, 100, 100 and generate new number facts  	8 x 3 = 5 x 3 + 3 x 3 Begin to draw links with multiples of 10 and 100 through carefully selected variation: 3 x 8 = 24 3 x 80 = 240 3 x 800 = 2400 30 x 8 = 240 300 x 8 = 2400
Expanded 'grid' method Multiply 2 digit number with a 1 digit number TO x O	 	Drawing of PV counters or Dienes. 24 x 6  	 **120 +24 = 144**

<table>
<tr>
<td>Applying

Conceptual variation</td>
<td>Investigate the relationship between multiplication and division using bar model.
 
</td>
<td>To strengthen the link between division and multiplication, and to present a varied form, the use of blank number grids is used to increase speed and mental agility whilst referring to the grid format.
 
</td>
<td>Balancing calculations (represented by scales) to show commutative law.

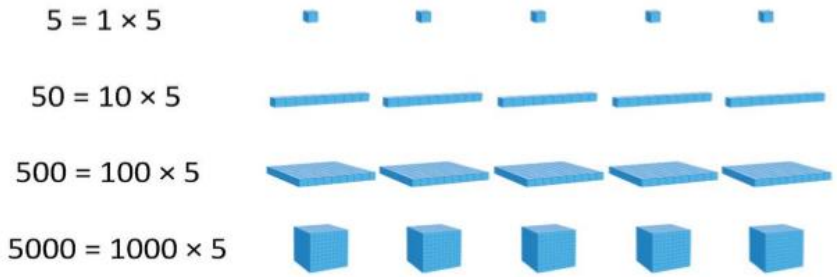
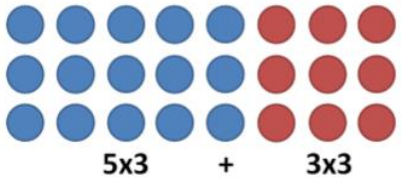
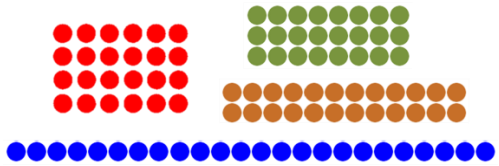
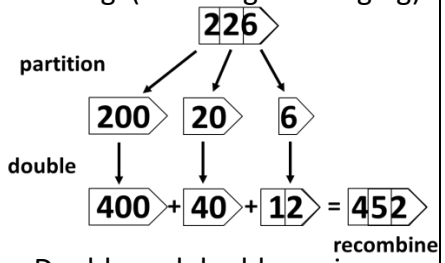
Solve problems involving missing number problems involving multiplication including positive number scaling problems and correspondence problems where n objects are connected to m objects.</td>
</tr>
</table>

Statutory Requirements:

- ⊗ Use place value, known and derived facts to multiply mentally, including x0 x1 and multiplying together three numbers
- ⊗ Recognise and use factor pairs and commutativity in mental calculations
- ⊗ Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- ⊗ Solve problems involving multiplying, including the distributive law to multiply two-digit numbers by one-digit including positive number scaling problems and correspondence problems where n objects are connected to m objects
- ⊗ **Time tables - Pupils recall all times tables up to 12 x 12.**

Vocabulary

odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), total, counting in 10s, 100s, 1000s, how many times? multiple of, multiply, multiply by, scale up, distributive law, regrouping, times tables, product of

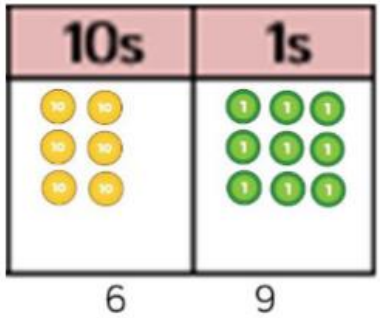
Objective	Concrete and Visual representations	Imagery	Abstract
Generate new multiplication facts using known fact	 * Pupils reinforce x10, x100 and x 1000 through conversions of units of measure in contextual situations.	Distributive law of multiplication 	8 x 3 = 5 x 3 + 3 x 3 Begin to draw links with multiples of 10 and 100 through carefully selected variation: 3 x 8 = 24 3 x 80 = 240 3 x 800 = 2400 30 x 8 = 240 300 x 8 = 2400
Mental strategies Including memorising **all times tables.**	**<u>Find factors of numbers up to 40</u> using arrays (e.g. 24)**  24 is therefore a multiple of 1, 2, 3, 4, 6, 8, 12 and 24 and these numbers are called its factors. They are numbers that will go into 24 without leaving a remainder, and they come in pairs, e.g. 6 and 4.	Double 3 digit numbers by partitioning (including exchanging)  X 4 = Double and double again. X 8 by doubling it three times	2 x 113 = 226 4 x 113 = 452 8 x 113 = 904

Formal method
Multiply 2 digit number with a 1 digit number

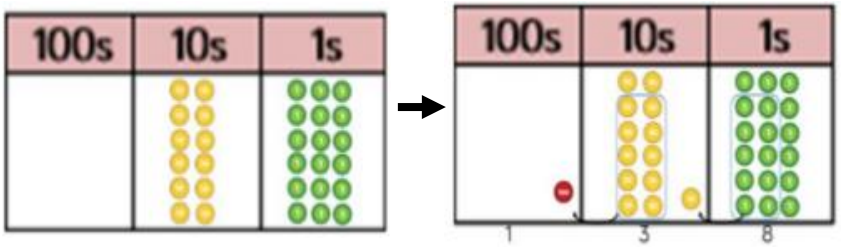
TO x O
<u>without exchanging</u>

TO x O
<u>with exchanging</u>

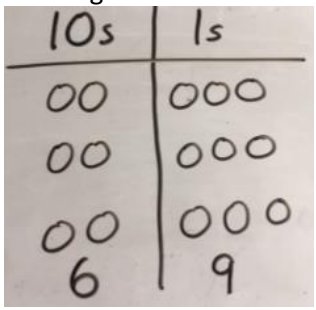
23 × 3 = (Estimate >60)

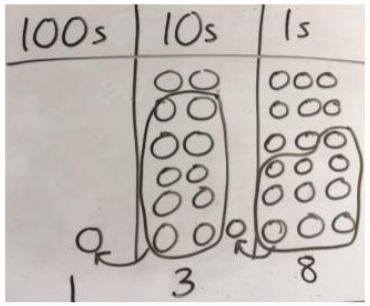


23 x 6 (Estimate >120)



Drawing of PV counters.





T O
2 3
X 3
0 9 (3 x 3)
6 0 (20 x 3)
6 9

(expanded) → (compact)

23
× 3
69

In the early stages of multiplying 1 digit numbers by 2 and 3 digit numbers use this expanded method.

H T O
2 3
X 6
1 8 (3 x 6)
1 2 0 (20 x 6)
1 3 8

(expanded) → (compact)

23
× 6
138
1 1

Formal method
Multiply 3 digit number with a 1 digit number

H T O x O
<u>with exchanging</u>

423 x 6 (estimate > 2400)

Children can use base ten apparatus to set out as above, or they can associate each section of the array as an area, as in this 'area model'

6	400	20	3
	2400	120	18

Children observe and describe **'What's the same and what's different'?** among these models

Draw the exchange on to the area model:

6	400	20	3
	2400	120	18
	100	10	

Th H T U
4 2 3
× 6
1 8
1 2 0
2 4 0 0
2 5 3 8

Th H T O
4 2 3
X 6
2 5 3 8
2 1 1

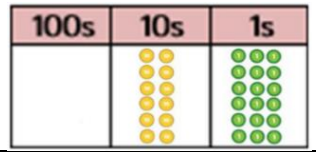
'What's the same and what's different'?

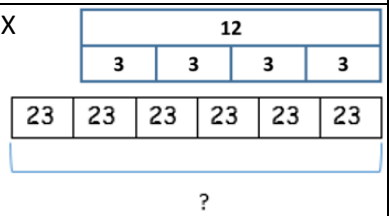
Applying

Conceptual Variation

⊗ Investigate the relationship between X and ÷ using bar model.

⊗ What's the calculation? What is the product?





6 x 7 =
6 x 70 =
6 x 700 =

60 x 7 =
? = 600 x 7
0.6 x 7 =

⊗ Find product of 6 and 23.
⊗ 6 x 23 =
⊗ ? = 23 x 6

⊗ Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

⊗ With the counters, prove that 6 x 23 = 138

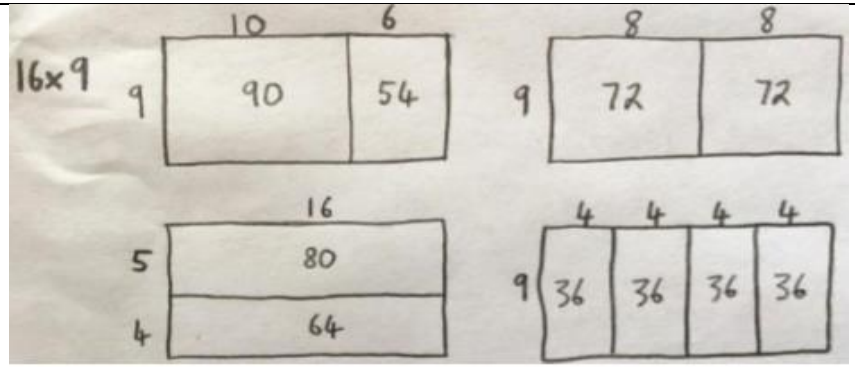
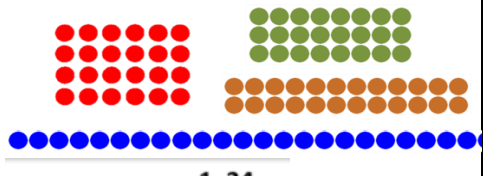
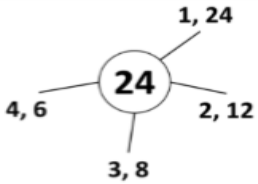
Statutory Requirements:

- ⊗ identify multiples and factors: all factor pairs of a number, common factors of two numbers
- ⊗ Establish whether a number up to 100 is prime and recall prime numbers up to 19
- ⊗ Recognise and use square numbers and cube numbers and their notation
- ⊗ Multiply numbers up to four digits by a one- or two-digit number using a formal written method
- ⊗ Multiply whole numbers and those involving decimals by 10, 100 and 1000.
- ⊗ Solve problems using multiplication and division using my knowledge of factors and multiples, squares and cubes

Vocabulary

odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), (common) multiple of, times, multiply, multiply by, scale up, decimal point, decimal place, factors, square number, cube number, prime number, prime factors,

In year 5 the children begin to multiply bigger numbers and progress to multiplying by 2 digits. However, it is entirely appropriate to revisit and check methods from Y4 to ensure that their place value knowledge and understanding of carrying is secure

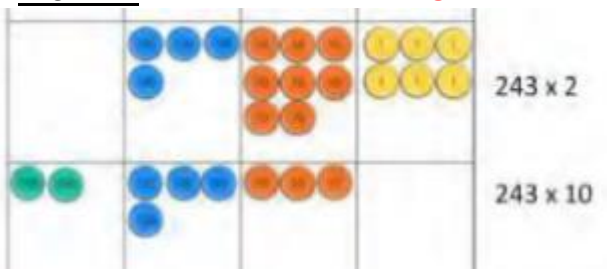
Objective	Concrete and Visual representations	Imagery	Abstract
Mental strategies Including memorising all times tables.	 'Area model' used to show multiplication where numbers are partitioned in different ways	**Find factors of numbers up to 40 using arrays (e.g. 24)**  	**Use times tables knowledge to find common multiples** 
Short multiplication	At all stages check that the children understand the process of carrying. **If necessary revisit with the use of concrete resources.**	7 × 342 300 40 2 2100 280 14 200 10	Th H T O 3 4 2 X 7 2 3 9 4 2 1

<table>
<tr>
<td>Use grid method to multiply pairs of 2-digit numbers</td>
<td>
<table>
<tr><th></th><th>30</th><th>4</th><th></th></tr>
<tr><td>20</td><td>600</td><td>80</td><td>= 680</td></tr>
<tr><td>3</td><td>90</td><td>12</td><td>= 102</td></tr>
</table>
Area model used in stead of PV counters because of the size of the numbers.
Green = ones
Yellow = tens
</td>
<td>Children can draw the Area Model to represent the multiplication.

Estimate by rounding to check the answer. (E.g. > 600 but less than 900)</td>
<td>
<table>
<tr><td>×</td><td>30</td><td>4</td><td></td></tr>
<tr><td>20</td><td>600</td><td>80</td><td>680</td></tr>
<tr><td>3</td><td>90</td><td>12</td><td>102</td></tr>
<tr><td></td><td></td><td></td><td>782</td></tr>
</table>
</td>
</tr>
<tr>
<td>Formal method (extended) long multiplication between a pair of 2 digit numbers.

TO x TO</td>
<td><u>74 X 63 (Estimate over 4200 but less than 4800)</u>

Start with this extended method before compact method.

Children should initially <u>write the calculations used down at the side. Draw attention to zeros as place holders</u></td>
<td>
Th H T O
7 4
x 6 3
1 2 (3 x 4)
2 1 0 (3 x 70)
2 4 0 (60 x 4)
4 2 0 0 (60 x 70)
4 6 6 2
</td>
<td>
Th H T O
7 4
X 6 3
2 2 2 (74 x 3)
1
+ 4 4 4 0 (74 x 60)
2
4 6 6 2
</td>
</tr>
<tr>
<td>Use long multiplication to multiply a 3-digit numbers by a 2digit number (less than 20)</td>
<td><u>243 X 12</u> H T O
 

Use place value counters then move on to the area model.</td>
<td>
<table>
<tr><td></td><td></td><td>200</td><td>40</td><td>3</td></tr>
<tr><td>2</td><td></td><td>400</td><td>80</td><td>6</td></tr>
<tr><td>10</td><td>2000</td><td>400</td><td>30</td><td></td></tr>
</table>
</td>
<td>
Th H T O
2 4 3
X 1 2
4 8 6 (x 2)
2 4 3 0 (x 10)
2 9 1 6
1
</td>
</tr>
<tr>
<td><u>3d × 3d</u> and <u>4d × 2d</u></td>
<td><u>When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract.</u></td>
<td></td>
<td></td>
</tr>
</table>

Statutory Requirements:

- ⊗ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- ⊗ Perform mental calculations, including with mixed operations and large numbers
- ⊗ Identify common factors, common multiples and prime numbers.
- ⊗ Use knowledge of the order of operations to carry out calculations involving the 4 operations
- ⊗ Multiply one-digit numbers with up to 2 decimal places by whole numbers
- ⊗ Solve problems involving multiplication and division which require answers to be rounded to specified degrees of accuracy

Vocabulary

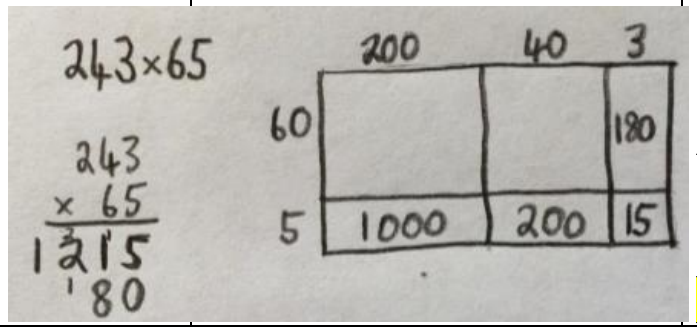
odd, even, double, halves, the same, lots of, groups of, times (once, twice etc.), (common) multiple of, times, multiply, multiply by, scale up, decimal point, decimal place, factors, square number, cube number, prime number, prime factors,

In year 6 the children begin to multiply bigger numbers and progress to multiplying by 2 digits. However, it is entirely appropriate to revisit and check methods from Y4, Y5 to ensure that their place value knowledge and understanding of carrying is secure.

Objective	Concrete and Visual representations	Imagery	Abstract
Mental strategies Including memorising all times tables.			
Formal method Use long multiplication to multiply a up to 5 digit numbers by a 2 digit number	When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract. However, imagery and concrete resources can be used to reason and explain the method.		

2 100 000		700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000		70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000		7000 x 3	700 x 30	70 x 300	7 x 3000		
2100		700 x 3	70 x 30	7 x 300			
210		70 x 3	7 x 30				
21	=	**7 x 3**					
2.1		0.7 x 3	7 x 0.3				
0.21		0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021		0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

Multiplication facts from a known fact.



	2	3	1	4
	X		2	3
	6	9	4	2
4	6	2	8	0
5	3	2	2	2

Reinforce zero as a place holder

<table>
<tr>
<td>Formal method
Multiply a decimal number by a 1 digit number</td>
<td>When multiplying decimals, <u>initially begin by using the extended method.</u>

As confidence grows children may find other strategies <u>e.g. multiply by 10/100 to eliminate decimal point and then adjust the answer.</u></td>
<td>
<table>
<tr><td>×</td><td>£20</td><td>£3</td><td>60p</td><td>7p</td><td></td></tr>
<tr><td>3</td><td>£60</td><td>£9</td><td>£1.80</td><td>21p</td><td>= £71.01</td></tr>
</table>
</td>
<td>
<table>
<tr><td></td><td>T</td><td>O</td><td>t</td><td>h</td></tr>
<tr><td>£</td><td>2</td><td>3</td><td>6</td><td>7</td></tr>
<tr><td>X</td><td></td><td>3</td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td></td><td>1</td><td>2</td><td>2</td><td></td></tr>
</table>
</td>
</tr>
<tr>
<td>Formal method
Long multiplication to multiply 3-digit then 4-digit numbers with decimals by numbers between 10 and 35;</td>
<td>36.21 x 17

Use rounding to approximate.
Estimate the answer is about less than 36 x 20 = 720</td>
<td></td>
<td>
<table>
<tr><td></td><td></td><td>3</td><td>6</td><td>.</td><td>2</td><td>1</td></tr>
<tr><td>x</td><td></td><td>1</td><td>7</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>4</td><td>1</td><td></td><td></td><td></td></tr>
<tr><td></td><td>2</td><td>5</td><td>3</td><td>.</td><td>4</td><td>7</td></tr>
<tr><td></td><td>3</td><td>6</td><td>2</td><td>.</td><td>1</td><td>0</td></tr>
<tr><td></td><td>1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>1</td><td>5</td><td>.</td><td>5</td><td>7</td></tr>
</table>
</td>
</tr>
<tr>
<td>Applying

Conceptual variation</td>
<td>
Long multiplication
24 × 16 becomes
<table>
<tr><td></td><td>2</td><td></td></tr>
<tr><td></td><td>2</td><td>4</td></tr>
<tr><td>×</td><td>1</td><td>6</td></tr>
<tr><td>2</td><td>4</td><td>0</td></tr>
<tr><td>1</td><td>4</td><td>4</td></tr>
<tr><td>3</td><td>8</td><td>4</td></tr>
</table>
Answer: 384

124 × 26 becomes
<table>
<tr><td></td><td>1</td><td>2</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td>4</td></tr>
<tr><td>×</td><td></td><td>2</td><td>6</td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>0</td></tr>
<tr><td></td><td>7</td><td>4</td><td>4</td></tr>
<tr><td>3</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>1</td><td></td><td></td></tr>
</table>
Answer: 3224

124 × 26 becomes
<table>
<tr><td></td><td>1</td><td>2</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td>4</td></tr>
<tr><td>×</td><td></td><td>2</td><td>6</td></tr>
<tr><td></td><td>7</td><td>4</td><td>4</td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>0</td></tr>
<tr><td>3</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>1</td><td></td><td></td></tr>
</table>
Answer: 3224

Short multiplication
24 × 6 becomes
<table>
<tr><td></td><td>2</td><td>4</td></tr>
<tr><td>×</td><td></td><td>6</td></tr>
<tr><td>1</td><td>4</td><td>4</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>
Answer: 144

342 × 7 becomes
<table>
<tr><td></td><td>3</td><td>4</td><td>2</td></tr>
<tr><td>×</td><td></td><td></td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>9</td><td>4</td></tr>
<tr><td></td><td>2</td><td>1</td><td></td></tr>
</table>
Answer: 2394

2741 × 6 becomes
<table>
<tr><td></td><td>2</td><td>7</td><td>4</td><td>1</td></tr>
<tr><td>×</td><td></td><td></td><td></td><td>6</td></tr>
<tr><td>1</td><td>6</td><td>4</td><td>4</td><td>6</td></tr>
<tr><td></td><td>4</td><td>2</td><td></td><td></td></tr>
</table>
Answer: 16 446
</td>
<td>Be aware of how calculations may be in a different order or presented differently.
What do you notice is the same/different?</td>
<td></td>
</tr>
</table>

RECEPTION	DIVISION and FRACTION

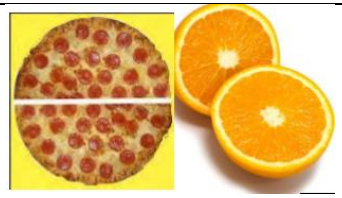
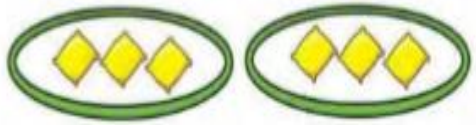
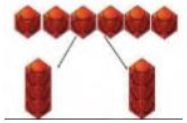
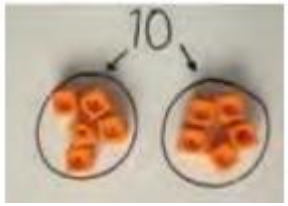
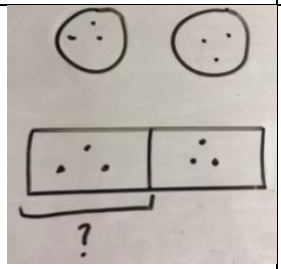
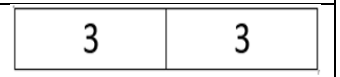
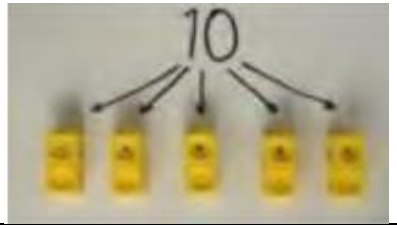
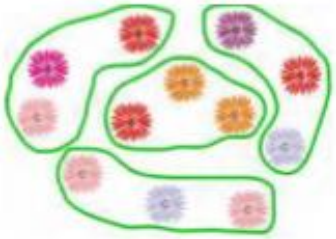
Statutory Requirements:

- Early Learning Goal - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing

Vocabulary

odd, even, double, halves, the same, lots of, groups of, share – share equally, divided by, left, left over

Early division should be introduced in EYFS predominately using language such as halving and sharing. To develop an understanding of the concepts children should use concrete resources and see representations of division as both grouping and sharing.

Objective	Concrete and Visual representations	Imagery	Abstract
Understand halving as dividing into 2 equal parts.	 		
Understand division as sharing	  Share a quantity into 2 equal halves using objects.  	Draw pictures of sharing a quantity into 2 equal parts. 	
Understand division as grouping	 Mum had 6 socks. She grouped them into pairs. How many pairs did she have? Now try again with 10 socks. Other scenario can be put pupils in groups of … how many group do we have? 	 Put flowers in groups of 3, how many groups?	

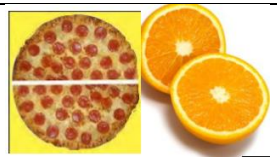
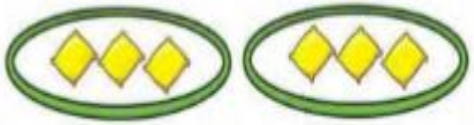
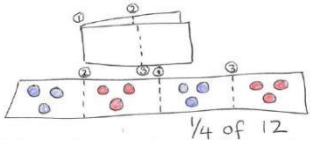
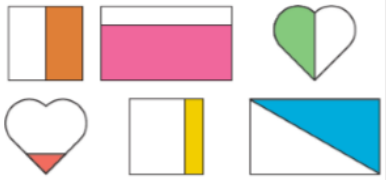
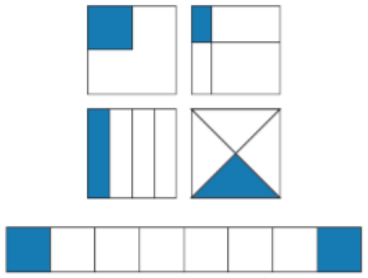
Statutory Requirements:

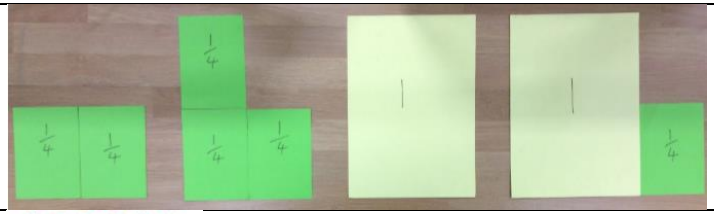
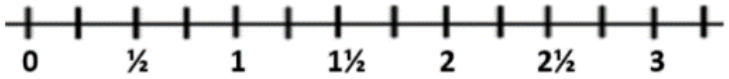
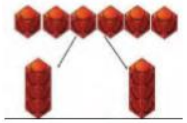
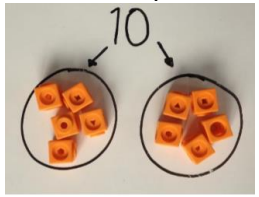
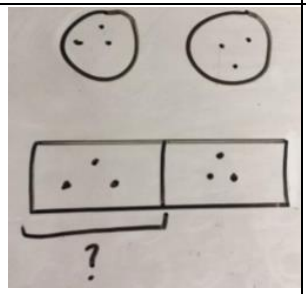
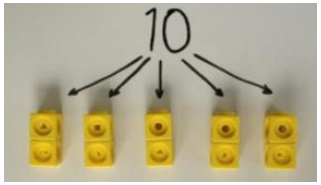
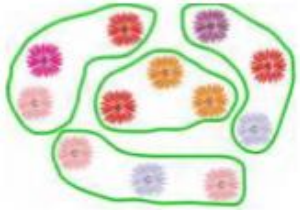
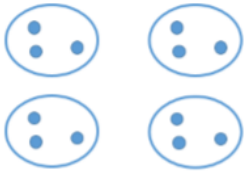
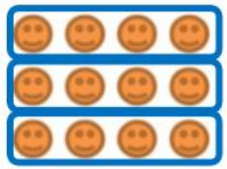
- ÷ Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- ÷ Recognise, find and name: one half (½) as one of 2 equal parts of an object, shape or quantity.
- ÷ Recognise, find and name: one quarter (¼) as one of 4 equal parts of an object, shape or quantity.

Vocabulary

odd, even, double, halves, **quarter, three quarters,** the same, lots of, groups of, share equally, **bar, altogether, divide, split, array** divided by, left, left over

Continue from EYFS, Y1 children continue to develop an understanding of the concepts using concrete resources and imagery representations of division as both grouping and sharing. Working with numbers up to 20.

Objective	Concrete and Visual representations	Imagery	Abstract
Understand halving as dividing into 2 equal parts.	 		What is half of this amount? 
Recognise and make one-half ($\frac{1}{2}$) in a range of ways (discern examples from non-examples); identify one quarter ($\frac{1}{4}$)	÷ Half ($\frac{1}{2}$), quarter ($\frac{1}{4}$) of a shape/capacity/length: fold papers, shapes, strings, playdough in half, then fold in half again to create quarters  ÷ Share number of objects into 2 equal parts to find ½ ,4 equal parts to find ¼ . Use 10-frame, and cubes.  ÷ Using a bar, pupils begin to explore **halving** and then subsequent **quarter**ing as a way of sharing and using a bar (piece of paper) folder in half to create two groups onto which items can be drawn or placed. This extends to quarters and sharing this into 4 groups. 	Which is ½  **Which of these diagrams are ¼ blue?** 	Colour half of each whole shape: 

Use halves and quarters as **counting** numbers, going over 1			
Understand division as **sharing**	 ÷ Physically **Share** objects into groups using circles, hoops or boxes.    ÷ Distribute into a divided bar. 	Draw pictures of sharing a quantity into 2 equal parts. *6 shared equally by 2 is 3.* Draw images to solve problem 	6 ÷ 2 = 3 3 | 3
Understand division as **grouping**	  Physically group items and count in groups. Socks, cubes… Other scenario can be put pupils in groups of x. How many group do we have?  *"There are x altogether."* *"There are x groups."* *"There are x in each group."*	 Solve division problems by drawing dots/flowers. Ring the dots to group flowers in groups of 3, how many groups?	6 ÷ 2 = 3 6 2 | 2 | 2 6 ÷ 2 = □ □ = 6 ÷ 2 6 ÷ □ = 3 3 = 6 ÷ □ □ ÷ 2 = 3 3 = □ ÷ 2 □ ÷ □ = 3 3 = □ ÷ □
Understand x and ÷ are opposite. Use **arrays** and **bar model** to represent division	 Sharing 12 in 4 equal groups, Thre are 3 in each group 4 lots of 3 make 12.  12 put in groups of 4, there're 3 groups. 3 equal groups of 4 make 12.	Solve 1 step problems using practical and drawing of groups , arrays and bar model. 12 3 | 3 | 3 | 3	4 x 3 = 12 3 x 4 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3

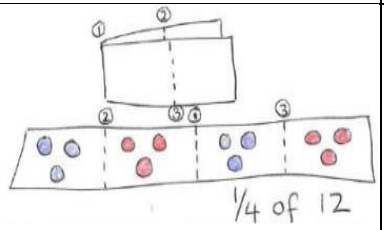
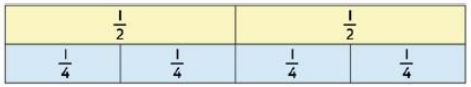
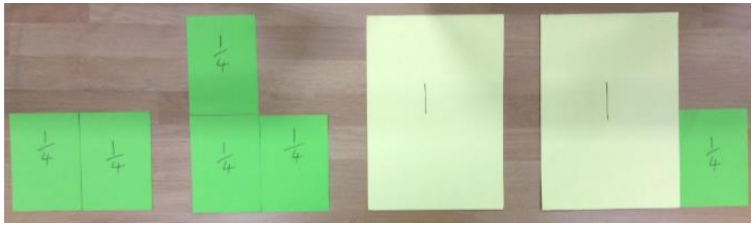
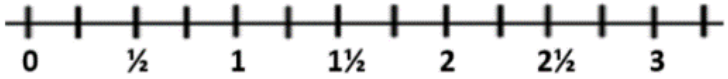
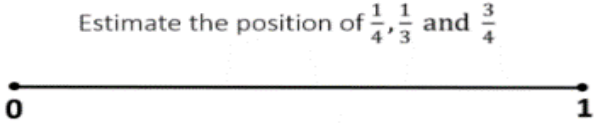
Statutory Requirements:

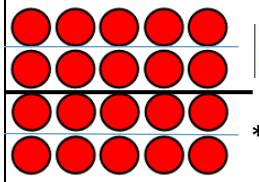
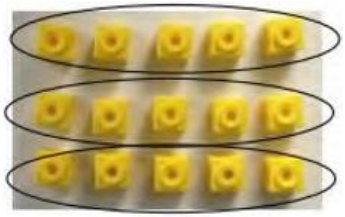
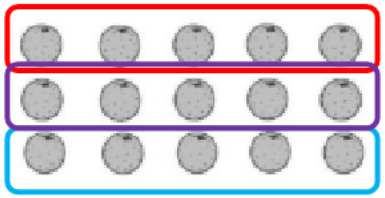
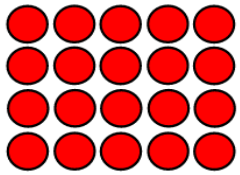
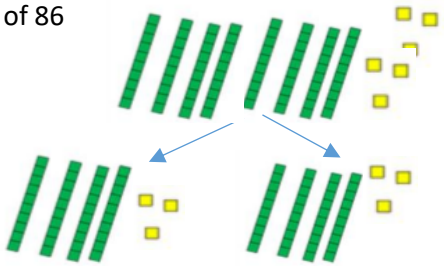
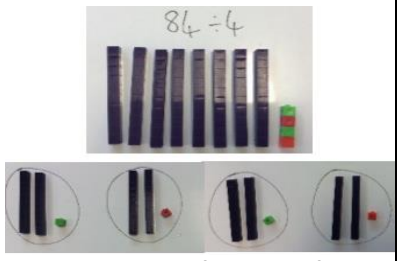
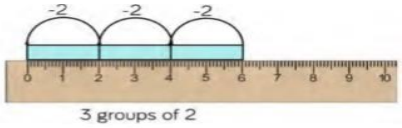
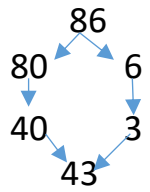
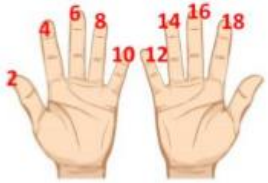
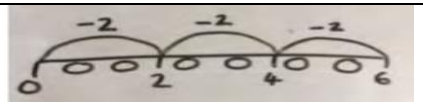
- ÷ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables fluently, including recognising odd and even numbers.
- ÷ Calculate mathematical statements for multiplication and division within the 2, 5 and 10 multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs
- ÷ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- ÷ Recognise, find, name and write 1/3, ¼ , 2/4, ¾ of a length, shape, set of objects or quantity
- ÷ Write simple fractions : ½ of 6=3
- ÷ Recognise equivalent fractions 2/4=½.

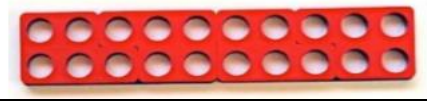
Vocabulary

odd, even, double, halves, quarter, three quarters, the same, lots of, groups of, share equally, bar, altogether, divide, split, array divided by, left, left over **division, chunks, multiples, fraction**

Follows on from the learning in year 1 with multiplication and is again linked to the use of arrays to develop greater conceptual understanding. The focus should be around using divisors of 2,5 and 10 (before progressing to 3).

Objective	Concrete and Visual representations	Imagery	Abstract
Compare ½ and ¼, learning that 2/4 = ½	Fold a strip of paper in half and then in half again. Label each equal section as ¼. Observe and describe the relationship between ¼ and ½. **Can you prove that two quarters (2/4) is the same as a half?** 		
Use halves and quarters as **counting** numbers, going over 1. Find ¾ of shapes and numbers		 The children can have ¾ of the cupcakes.  	

<table>
<tr>
<td>Find ½ , ¼ of any numbers less than 40 using arrays, bar model, sharing, grouping.
Use sharing and grouping for division.
Relate division to finding unit fractions of quantities</td>
<td><u>Practically</u> <u>sharing</u> or <u>grouping</u> objects to find a ½ or a ¼ of an amount less than 40. Encourage children to arrange the objects in lines or arrays as they group and share
  
* Notice that ÷ 2 is the same as a <u>half</u> and ÷ 4 is the same as a <u>quarter.</u>
 
Find 15 ÷ 3 or 15÷5
Put cubes in arrays, group 5 cubes in each row. There're 3 groups.
Or share 15 cubes into 3 (lines). There are 5 in each line.</td>
<td><u>O O O O O O O O O O O O O</u>
O O O O O O O O O O O O O
Draw array to find half (e.g. of 24)
Or quarters O O O O O
O O O O O
O O O O O
O O O O O
 
Find 15 ÷ 3 or 15÷5</td>
<td>Record
½ of 20 = 10
¼ of 20 = 5
Children can generate number facts from known number. Eg. Half of 6 is 3 so half of 6 tens is 3 tens. Ect.
15 ÷ 5 = 3
15 ÷ 3 = 5</td>
</tr>
<tr>
<td>Use arrays and bar model to represent the inverse</td>
<td>Revise this inverse concept from Y1. Ask children to comment on similarities and differences between these 2 models, using appropriate vocabulary
  
Solve 1 step problems using practical and drawing of groups , arrays and bar model.</td>
<td><table><tr><td colspan="4">20</td></tr><tr><td>5</td><td>5</td><td>5</td><td>5</td></tr></table></td>
<td>? x ? = 20
20 = ? x ?
20 ÷ ? = ?
? ÷ ? = ?</td>
</tr>
<tr>
<td>Mental strategy
Find half of 2 digit numbers by partitioning (no exchanging)</td>
<td>Half of 86  Quarter of 84 
Pupils show that when halving an odd number, a remainder of one is left.</td>
<td>Drawing of dienes to show the practical sharing.
 
3 groups of 2</td>
<td> 
¼ of 84 by halving it twice.</td>
</tr>
<tr>
<td>Division on a number line. (chunking)</td>
<td> 
Explore division as grouping with bead bar or cubes in a row and move towards showing this as a number line: Use fingers to count in 2s.
1. Count in multiples on your fingers
2. Stop at the large number
3. How many fingers is the answer?</td>
<td> 
Using a number line with marked divisions, count back under the number line to show the groups of 2: number 'chunks.'</td>
<td>Represent this using written symbols and begin to show this as a written calculation:
2 x 9 = 18
18 ÷ 2 = 9.</td>
</tr>
</table>

Applying Conceptual Variation	**How many cars are needed to take 18 children to the match? 4 children per car.** 	Grouping strategy modelled with covered arrays and Numicon: how many [divisors] in [dividend]?  **20 ÷ 5 = 4** **20 dots.** **How many rows?** 	Missing number problems.

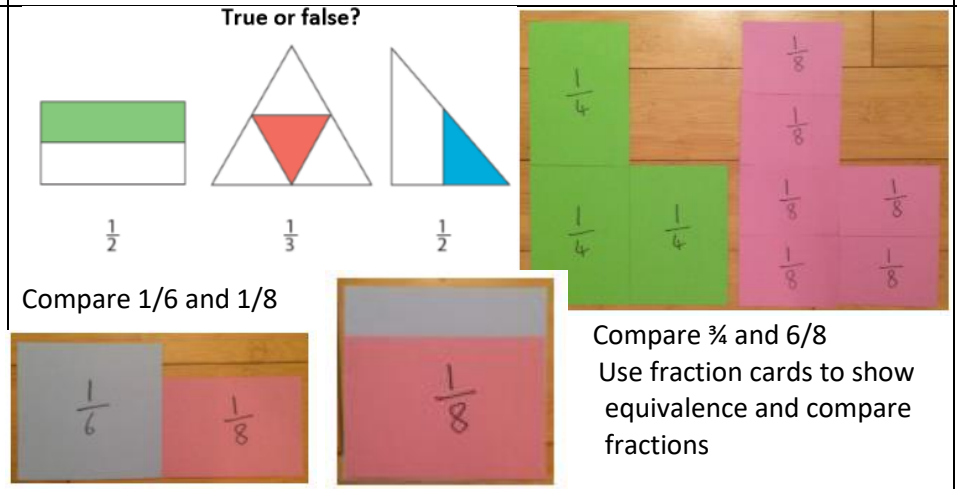
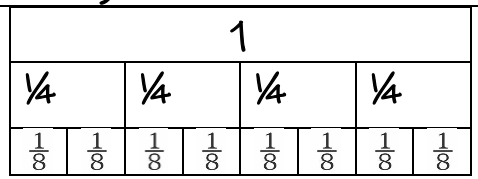
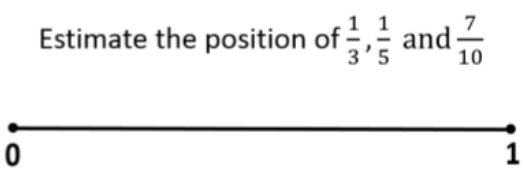
Statutory Requirements:

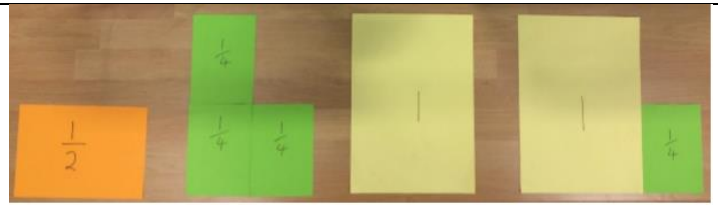
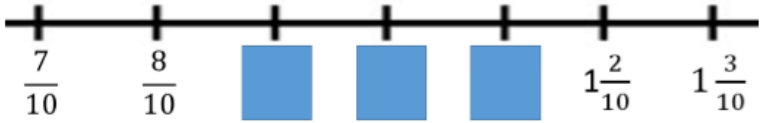
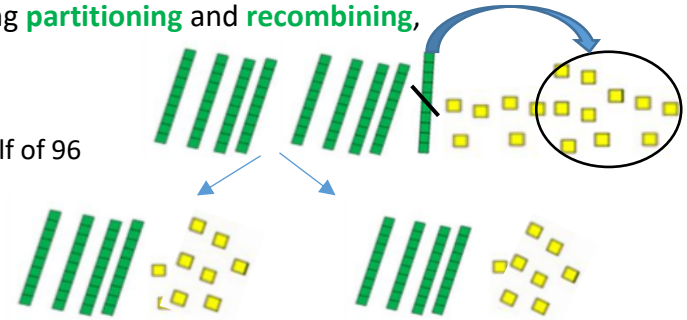
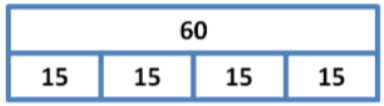
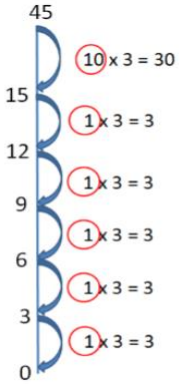
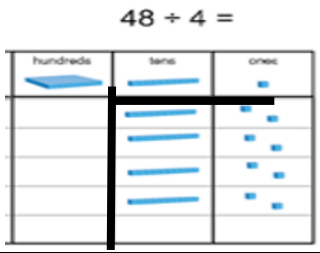
- ÷ Recall and use multiplication and division facts for the 3, 4 and 8 x tables
- ÷ Write and calculate mathematical statements for division using the multiplication tables they know, including 2-digit divided by 1-digit using mental and progressing to formal written methods
- ÷ Solve problems involving division, including missing number problems, or positive number scaling problems and correspondence problems where n objects are connected to m objects.
- ÷ Recognise, find and write fractions of a discrete set of objects using Unit and non-unit fractions with small denominators
- ÷ Recognise and show equivalent fractions
- ÷ Compare, order and +/- unit fractions, fractions with same denominators

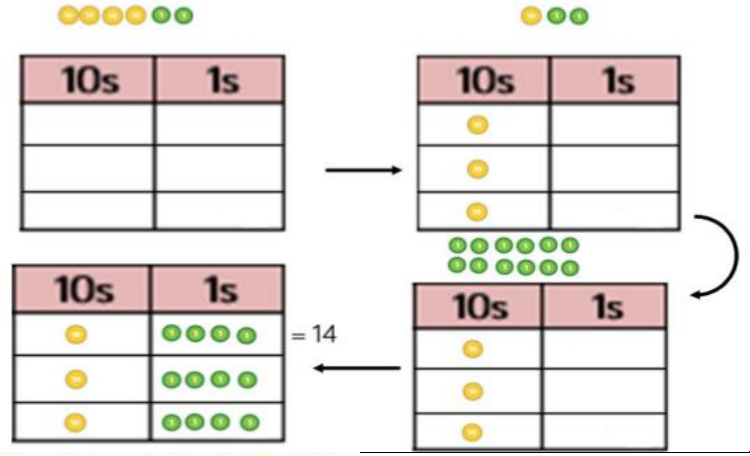
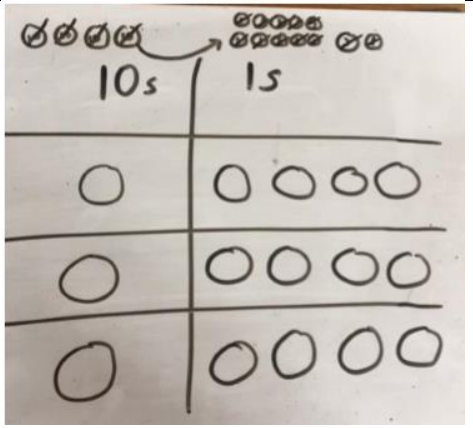
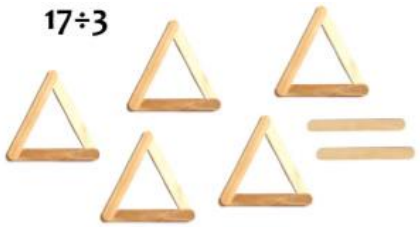
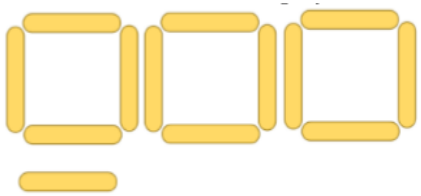
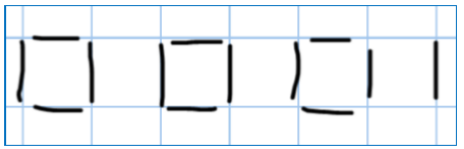
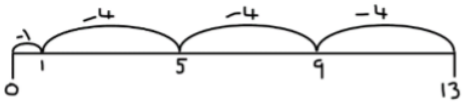
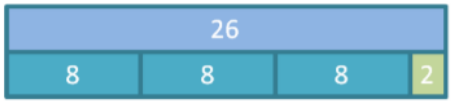
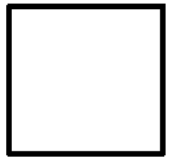
Vocabulary

odd, even, double, halves, quarter, three quarters, the same, lots of, groups of, share equally, bar, altogether, divide, split, array divided by, left, left over division, chunks, multiples, fraction **partitioning, recombining, divisor, dividend, quotient**

In order to access the curriculum at this stage it is essential that the children are developing fluency with their times tables as they will be beginning to work with a wider range of divisors: x2, x5, x10, x3, x4, x6, ,x8 and x9. Strategies should be supported heavily at the beginning with concrete resources and pictorial representations.

Objective	Concrete and Visual representations	Imagery	Abstract
Simple unit/non -unit fractions represented in a range of ways; Different fractions compared including equivalence			Identify fraction of shaded shape; position fractions on a number line; 

Counting in ¼, ½ and 1/10. Fractions as **counting numbers** going over 1.			
Finding half of a given number by partitioning (including exchanging)	Using **partitioning** and **recombining**, Half of 96 	Draw Dienes to accompany recording 96 80 + 16 (partition in a different way) 40 8 (half each part) 48 (Recombine)	½ of 96 = 48 Represent ½, ¼ using bar model **60 ÷ 4 = 15**  *'60 in four equal parts'*
Divide by 10 **Finding 1/10 (one tenth) of a number.**		Using a place value slider, pupils begin to explore how the **values of digits** change when dividing by 10.	
Chunking method (division on a number line) 2 digit number divided by a single digit	 **28 ÷ 7 = 4**  *'How many 7s in 28?'* Encourage children to use their times table knowledge.		45 ÷ 3 = □ □ x 3 = 45 10 x 3 = 30 15 5 x 3 = 15
Formal method **2d ÷ 1d** (no exchanging)		Draw the concrete base 10 resources	1 2 4 | 4 8

Formal method **<u>2d ÷ 1d</u>** (with exchanging)	3**42 ÷ 3**  	 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 Begin to record formally 1 4 3 ⟌ 4 ¹2
2digit number divided by 1 digit <u>with</u> **<u>remainders</u>**	 How many 3s are in 20? 17÷3  **13 ÷ 4**  Cuisenaire rods, above a ruler can also be used.	Children to represent the lollipop sticks pictorially.  '3 groups of 4, with 1 left over'   	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.
Applying Conceptual Variation	 	**Positive Integer Scaling** An example of a concrete problem here is where a simple drawing or amount can be '**scaled up**' or '**scaled down**' using an integer. Here is a square. Its sides are 12cm in length. Draw this shape 3 times smaller.  I have four £1 coins. How many do I have if I have eight times more.  	

YEAR 4 — DIVISION

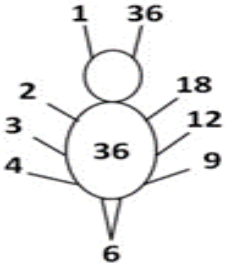
Statutory Requirements:

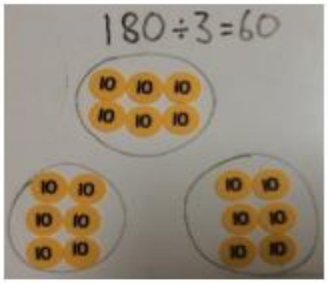
- ÷ Recall multiplication and division facts up to 12 x 12
- ÷ Use place value, known and derived facts to divide mentally, including dividing by 1
- ÷ Solve problems involving dividing a three-digit number by one-digit and number using a formal layout
- ÷ Recognise and show equivalent fractions
- ÷ Decimal equivalent of ¼, ½, ¾
- ÷ Compare, order and +/- unit and non-unit fractions, fractions with same denominators

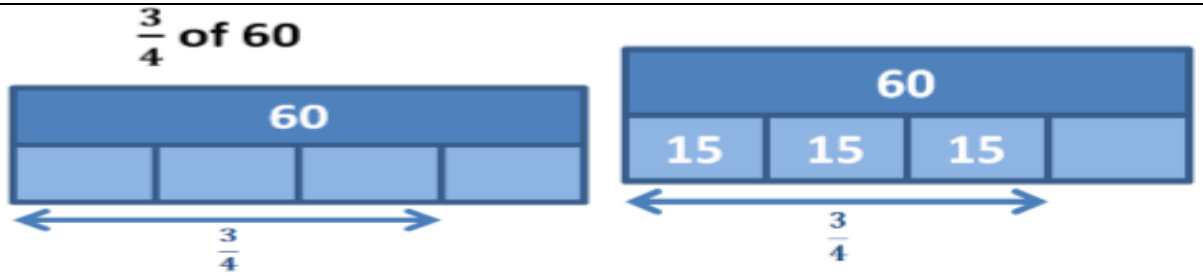
Vocabulary

odd, even, double, halves, quarter, three quarters, the same, lots of, groups of, share equally, bar, altogether, divide, split, array, left over, division, divided by, chunks, multiples, fraction partitioning, recombining, divisor, dividend, quotient, **short-division, algorithm, prime number, long-division, factor pairs, square**

Division in year 4 builds on the informal method taught in year 3. It is crucial that the children are becoming increasingly fluent with their times table knowledge and associated facts. As they begin to divide using bigger 'chunks' it is also essential that their place value knowledge is secure, in order to readily access the concepts involved.

Objective	Concrete and Visual representations	Imagery	Abstract
Find equivalent fractions, calculate fractions of amounts (unit and non-unit fractions	  $\frac{3}{4} = \frac{12}{16}$		
Find all factors pairs	Systematically, using times table knowledge. Record as the 'factor bug'		

<table>
<tr><td>Fraction of quantities shown using PV counters and bar models</td><td colspan="4"> 

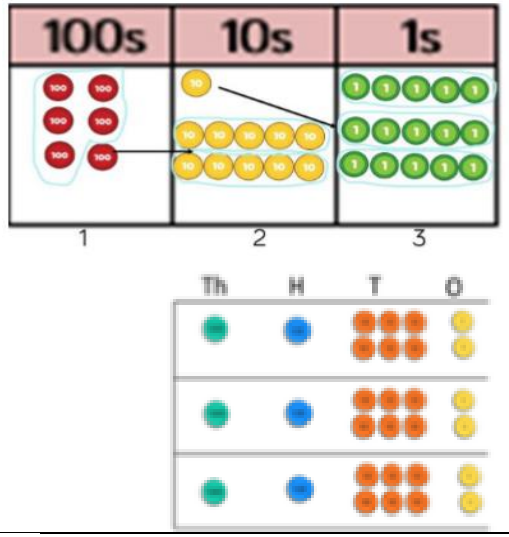
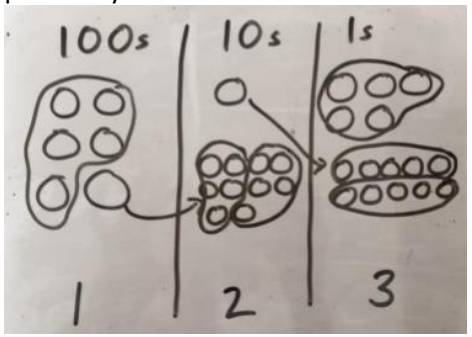

</td></tr>
<tr><td>Divide by 10, 100

Finding 1/10, 1/100 of a number.</td><td colspan="4">Using a place value slider, pupils begin to explore how the values of digits change when dividing by 100.
 
</td></tr>
<tr><td>Short division formal method

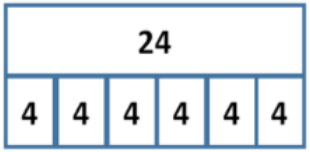
HTO ÷ O

ThHTO ÷ O</td><td>Using place value counters to group.
615 ÷ 5

How many groups of 5 hundreds can you make with 6 hundred counters?
Exchange 1 hundred for 10 tens.

3,486 ÷ 3</td><td> 
</td><td>Represent the place value counters pictorially.
 
</td><td>123
5 ⟌ 6 1 15

1 1 6 2
3 ⟌ 3 4 18 6</td></tr>
<tr><td>Applying

Conceptual Variation</td><td>This image shows 4 x 6
  </td><td>Change the image to show 4 x 7</td><td>This image shows 4 x 6
 

Use the image to calculate 4 x 12</td><td></td></tr>
</table>

Statutory Requirements:

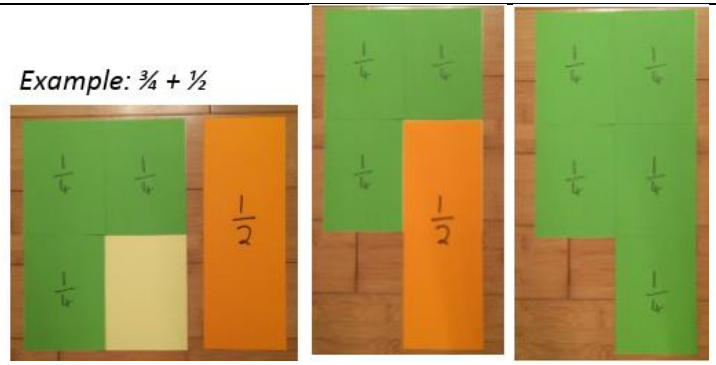
- Identify multiples and factors, including finding all factor pairs of a number, common factors of two numbers, know and use the vocabulary of prime numbers and establish whether a number up to 100 is prime
- Multiply and divide numbers mentally drawing on known facts
- Divide numbers up to 4 digits by a one-digit number using a written method and interpret remainders appropriately for the context
- Divide whole numbers and those involving decimals by 10, 100 and 1000. Read and write decimal numbers as fractions
- Compare and order fractions (with denominator is a multiples of same number)
- Identify, name and write equivalent fractions
- Recognise and convert between mixed numbers and improper fractions
- +/- fractions (denominator is a multiples of the same number)
- Multiply proper fractions and mixed numbers by whole numbers
- Recognise '%' and write percentages as a fraction (denominator '100') and a decimal
- Solve problems knowing decimal and % equivalents

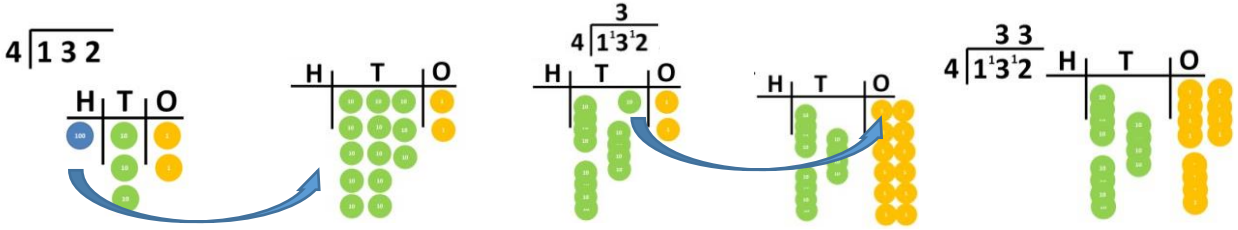
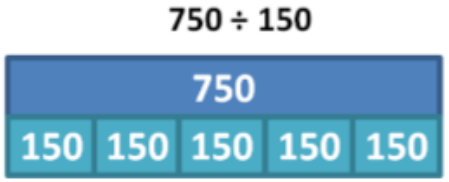
Vocabulary

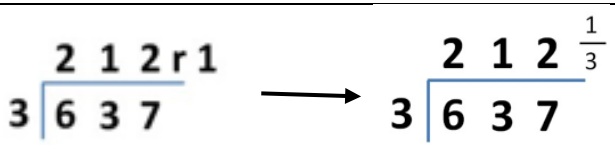
odd, even, double, halves, quarter, three quarters, the same, lots of, groups of, share equally, bar, altogether, divide, split, array, left over, division, divided by, chunks, multiples, fraction partitioning, recombining, divisor, dividend, quotient, short-division, algorithm, prime number, factor pairs, square, place value holder, **integer**

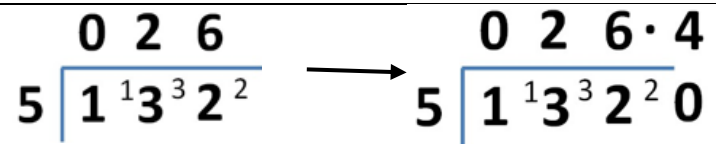
In year 5 the children will begin to work with bigger numbers and a wider range of divisors. They will need to spend time the challenging concepts taught in year 4, particularly with exchanging. It is appropriate to continue to use place value counters as a mechanism for support.

Objective	Concrete and Visual representations	Imagery	Abstract
Compare and order fractions, find equivalent fractions. +/- fractions	  Fraction cards used to compare, show equivalence and model calculations.		5/6 > 2/3 1/3 = 4/12 ¼ = 3/12 1 – 2/3 = 1/3 1 – 1/6 = 5/6

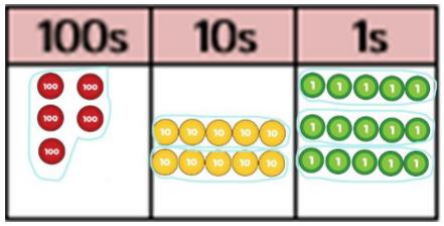
<table>
<tr><td>Add fractions</td><td>Example: ¾ + ½
 
 
Cuisenaire rods can be used as visual support along side or instead of fraction cards.</td><td>1/3 + ¼ = 4/12 + 3/12

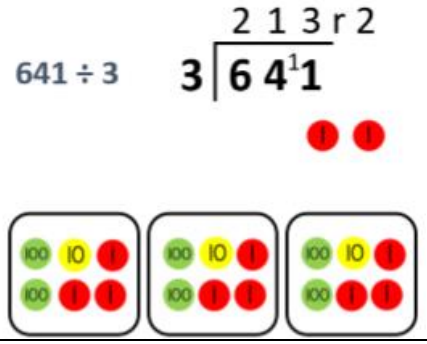
¾ + ½ = 5/4 = 1 ¼</td></tr>
<tr><td>Find decimal equivalents for quarters, fifths and tenths, relating to division.</td><td>Dividing length of a metre ruler into two/four/five equal parts.
  </td><td></td></tr>
<tr><td>Formal method
Short division without a remainder</td><td>Revise on short division, formal written method, introduced in Y4.
Exchange when/if needed and Group the place counters into groups of [divisors] 4 until all used up or a remainder is left. Show these groupings in the algorithm with the number of groups and the remainder.
 
Bar model used to reinforce ‘how many [divisors] in [dividend]?’
  </td><td></td></tr>
<tr><td>Short division with integer remainders</td><td>Draw PV counters to support recording</td><td>2 1 2 r 1
3 | 6 3 7</td></tr>
</table>

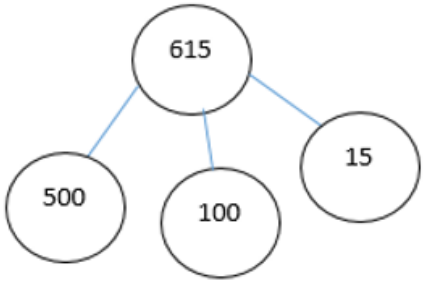
<table>
<tr>
<td>Short Division with a fraction remainder and decimal remainders</td>
<td colspan="2">


Record the remainder 1 whole being divided by 3 as 1/3.
</td>
<td colspan="2">


Use decimal place value counters here.
Exchange 2 ones with 20 tenths.
</td>
</tr>
<tr>
<td>Applying

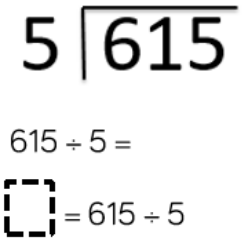
Conceptual Variation</td>
<td>
What is the calculation? What is the answer?




This is a diagram showing 641 ÷ 3. Can you explain it? What's the same? What's different between these 2 diagrams?
</td>
<td>
Using the part whole model below, how can you divide 615 by 5 without using short division?


</td>
<td>
I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

= 615 ÷ 5
</td>
<td>


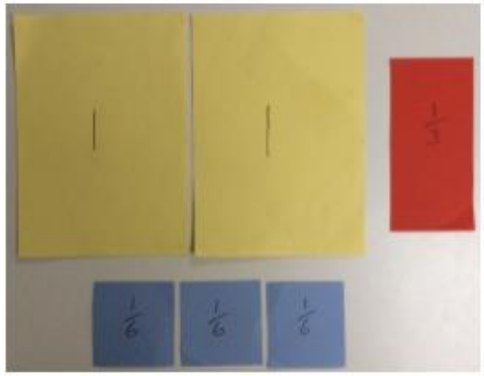
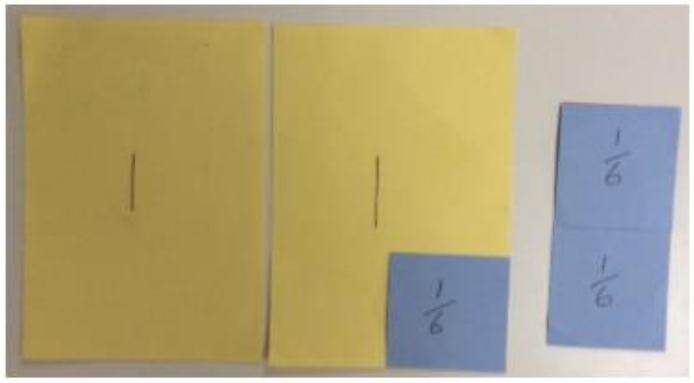
</td>
</tr>
</table>

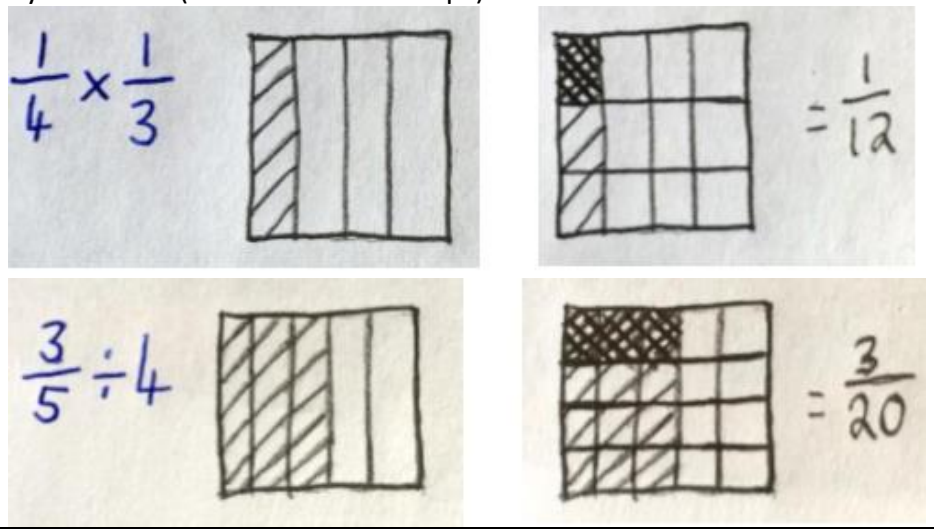
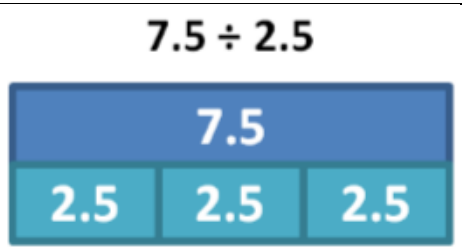
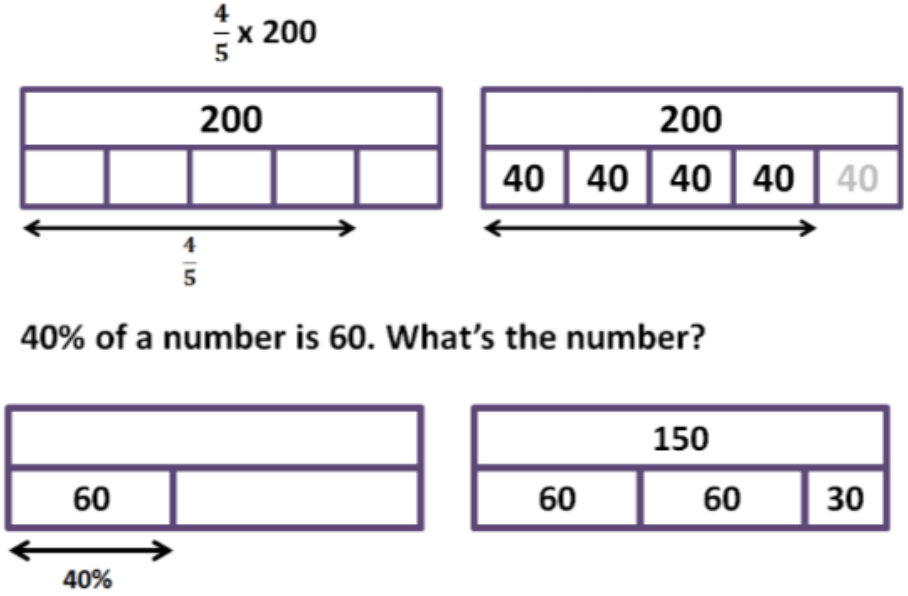
Statutory Requirements:

- ÷ Divide numbers up to 4 digits by a two-digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.
- ÷ Divide numbers up to 4 digits by a two-digit number using the formal written method of short division as appropriate.
- ÷ Use common factors to simplify fractions, common multiples to express fractions in same denominator
- ÷ Compare, order and +/- fractions including fractions>1 and fractions with different denominator
- ÷ x simple pairs of proper fractions (answer in simplest form)
- ÷ ÷ proper fractions by whole number.
- ÷ x/÷ nos by 10, 100, 1000
- ÷ Solve problems involving relative sizes of two quantities (missing values using integer x/÷ facts)
- ÷ Solve problems involving the calculation of % and the use of % for comparison
- ÷ Solve problems involving similar shapes where the scale factor is known or can be found
- ÷ Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

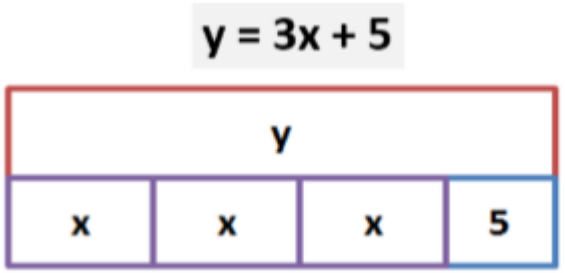
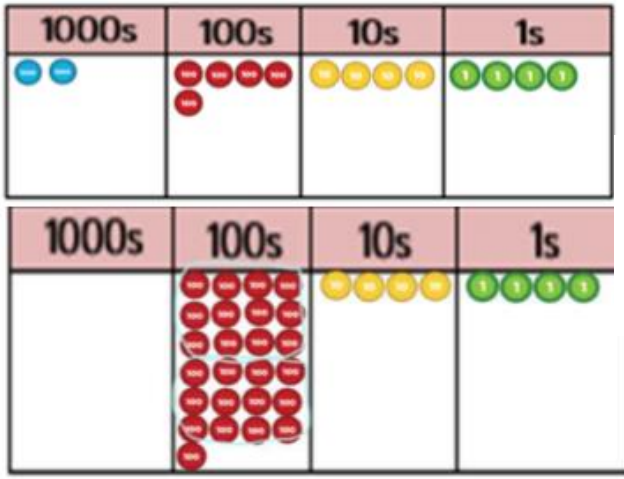
Vocabulary

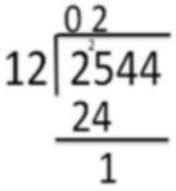
odd, even, double, halves, quarter, three quarters, the same, lots of, groups of, share equally, bar, altogether, divide, split, array, left over, division, divided by, chunks, multiples, fraction partitioning, recombining, divisor, dividend, quotient, short-division, **long division,** algorithm, prime number, factor pairs, square, place value holder, **integer**

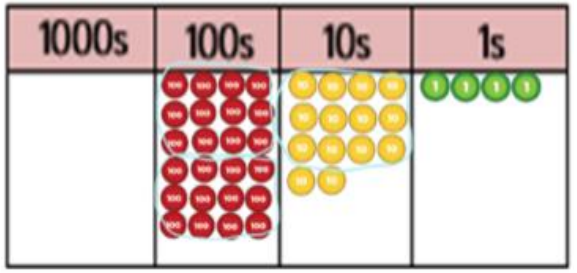
Objective	Concrete and Visual representations	Imagery	Abstract
Add and subtract fractions with different denominators	Fraction cards to show conversion into common denominators and calculating over whole-number boundaries.  		Example: $2\frac{1}{3} - \frac{3}{6}$

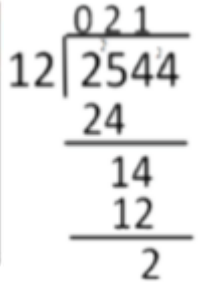
<table>
<tr>
<td>Multiply and divide unit fractions and simple non-unit fractions</td>
<td>Area model diagrams to model a fraction being divided or multiplied by a fraction (modelled in two steps).
 
</td>
<td> 
</td>
<td></td>
</tr>
<tr>
<td>Calculate percentages and fractions of quantities</td>
<td colspan="3">Bar model visualises finding fraction/percentage of quantity and finding the whole given a percentage/fraction. Shown step-by-step.
 
</td>
</tr>
</table>

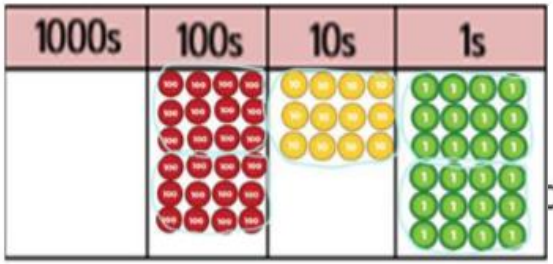
<table>
<tr><td>Describe linear number sequences, including using formulae in the form y = mx + c</td><td>Numicon and bar model used to model linear number sequences or equations.
 

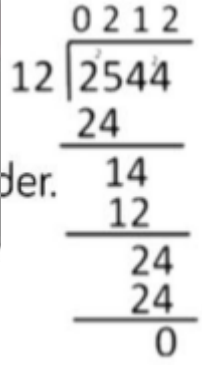
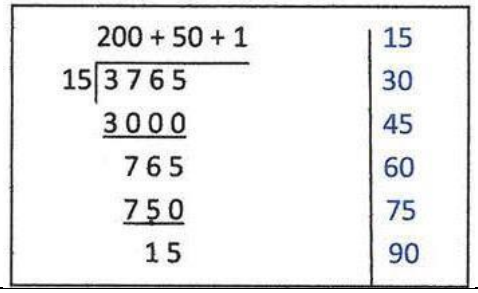
 
</td></tr>
<tr><td>Formal method
Long division</td><td> 

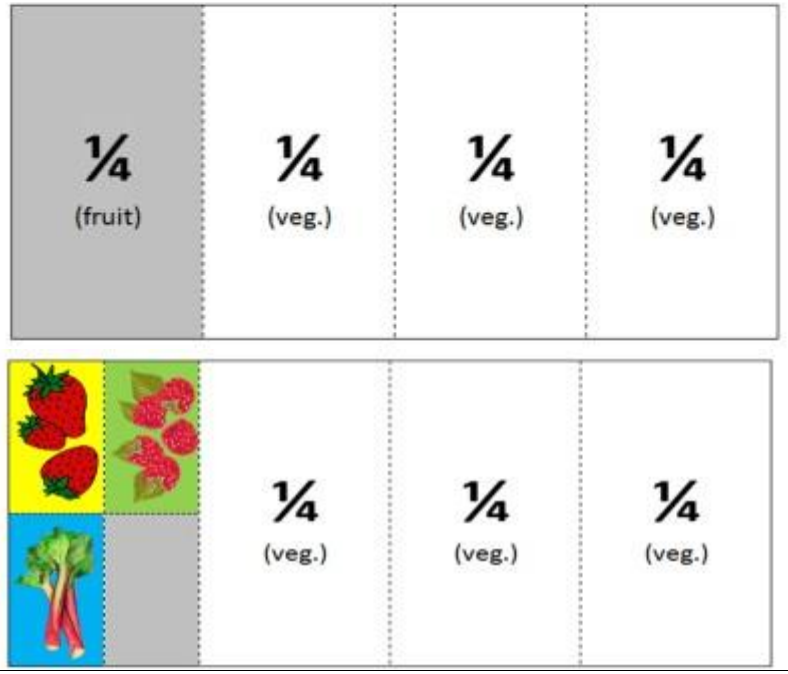
 

 
</td></tr>
<tr><td>Formal method
Long division</td><td> 

Note that in the quotient area, the separate steps are noted – similar to how chunking would be but more closely linked to the standard ‘look’ of this formal written strategy. This allows pupils to understand the value of the digits in the answer as they move through the calculation.</td></tr>
</table>

Long division with remainders	432 ÷ 15 becomes $\begin{array}{r} 28 \text{ r } 12 \\ 15\overline{)432} \\ 300 \\ \hline 132 \\ 120 \\ \hline 12 \end{array}$ Integer remainder	432 ÷ 15 becomes $\begin{array}{r} 28 \\ 15\overline{)432} \\ 300 \quad (15\times20) \\ \hline 132 \\ 120 \quad (15\times8) \\ \hline 12 \end{array}$ $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ Fraction remainder	432 ÷ 15 becomes $\begin{array}{r} 28.8 \\ 15\overline{)432.0} \\ 300 \\ \hline 132 \\ 120 \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$ Decimal remainder Answer: 28·8
Applying Conceptual Variation		***¼ of the plot is for fruit and ¾ of the plot is for vegetables. If ½ of the fruit area is for growing strawberries, what fraction of the whole plot is that?*** Draw a line to divide the ¼ into ½s and record ½ × ¼ = 1/8. ***¼ of fruit areas is for raspberries, the other ¼ for rhubarb. What fraction of the whole plot are each of these?*** Draw lines to show this. Record the calculation to show this. ¼× ¼ = 1/16. Extend to multiplying non unit fractions: ***½ of the vegetable area is to be given over for potatoes.*** Draw a horizontal line to divide this area in 2. Record ½ × ¾ = 3/8. Ask children what they notice happens when we multiply fractions together. Discuss how we multiply the numerators together and the denominators together to give us the answer!	